

COMPARISON THEOREMS IN SUB-RIEMANNIAN GEOMETRY

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The typical Riemannian comparison theorem is a result in which a *local* bound on the curvature (e.g. $\text{Ric} \geq \kappa$) implies a *global* comparison between some property on the actual manifold (e.g. diameter) and the same property on a constant curvature model. The generalization of these results to the sub-Riemannian setting is not straightforward, the main difficulty being the lack of a proper theory of Jacobi fields, an *analytic* definition of curvature and, a fortiori, constant curvature models.

Some comparison results, valid for 3D sub-Riemannian structures, have been recently obtained by Agrachev and Lee and generalized to contact manifolds with symmetries by Lee, Li and Zelenko. Building on these results, we develop a theory of Jacobi fields valid for *any* sub-Riemannian manifold, in which the Riemannian *sectional* curvature is generalized by the *canonical curvature* introduced by Agrachev and his students.

This allows to extend a wide range of comparison theorems to the sub-Riemannian setting. In particular, we focus on sectional and Ricci-type comparison theorems for the existence of conjugate points along sub-Riemannian geodesics. In this setting, the models with constant curvature are represented by Linear-Quadratic optimal control problems with constant potential. As an application, we prove a sub-Riemannian version of the Bonnet-Myers theorem and we obtain some new results on conjugate points for three dimensional left-invariant sub-Riemannian structures.

This is a joint work with D. Barilari (Paris 7).

References

1. D. Barilari, L. Rizzi Comparison theorems for conjugate points in sub-Riemannian geometry // arXiv:1401.3193