

LOCAL AND METRIC GEOMETRY OF NONREGULAR WEIGHTED CARNOT–CARATHÉODORY SPACES

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We investigate local and metric geometry of weighted Carnot–Carathéodory spaces in a neighbourhood of a nonregular point [8]. Such spaces are a wide generalization of classical sub-Riemannian spaces (which are smooth manifolds equipped by bracket-generating distributions of “horizontal” vector fields) and naturally arise in control theory (including cases when the dependence on control functions may be nonlinear), harmonic analysis, subelliptic equations etc.

For the spaces that we consider, there may be no analog of the intrinsic Carnot–Carathéodory metric (defined in sub-Riemannian geometry as the infimum of lengths of all “horizontal” curves joining the two given points) might not exist, and some other new effects, caused by the arbitrary weights of the vector fields, take place, which leads to necessity of introducing new methods of investigation of geometry of such spaces.

We describe the local algebraic structure of such a space, endowed with a natural quasimetric (first introduced by A. Nagel, E. M. Stein and S. Wainger in [5]) induced by the given weighted structure. We compare local geometries of the initial CC space and its tangent cone (which is a homogeneous space of a nilpotent Lie group) at some fixed (maybe nonregular) point.

Our considerations heavily rely on similar results about equiregular Carnot–Carathéodory spaces [4, 3] and adaptations of different “lifting” methods [6, 2, 1], which allow to reduce some questions about nonregular spaces to similar questions about the equiregular ones. Also, we use a generalisation to quasimetric spaces of the Gromov–Hausdorff spaces for metric spaces, which was constructed earlier in [7], and study new properties of the considered quasimetrics.

References

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