Analytical Properties of Sobolev Mappings on Roto-Translation Groups

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The roto-translation group, $SE(2)$, is a three-dimensional topological manifold diffeomorphic to $\mathbb{R}^2 \times \mathbb{S}^1$ with coordinates (x, y, θ). The left-invariant vector fields

$$X_1 = \cos\theta \frac{\partial}{\partial x} + \sin\theta \frac{\partial}{\partial y},\ X_2 = \frac{\partial}{\partial \theta},\ X_3 = -\sin\theta \frac{\partial}{\partial x} + \cos\theta \frac{\partial}{\partial y},$$

form a basis of the Lie algebra of $SE(2)$. The bracket-generating subbundle of the tangent-bundle is spanned by the frame X_1, X_2.

Consider the basic 1-forms dX_1, dX_2, dX_3, dual to the basic vector fields X_1, X_2, X_3, i.e., $dX_i(X_j) = \delta_{ij}$. Applying the methods developed in [1] we establish a key relation underlying the connection between mappings with bounded distortion [2] and nonlinear potential theory.

Theorem. *Let $SE(2)$ be a roto-translation group and $\Omega \subset SE(2)$ is an open set. Suppose that $f\colon \Omega \to SE(2)$ is a Sobolev mapping of the class $W^1_{4,\mathrm{loc}}(\Omega)$, $V\colon SE(2) \to \mathbb{R}^2$ is a vector field $V = (v_1, v_2) \in C^1$ such that $\mathrm{div}_h V = X_1 v_1 + X_2 v_2$ is bounded on $SE(2)$, and*

$$\omega(g) = v_1(g)\, dX_2 \wedge dX_3 - v_2(g)\, dX_1 \wedge dX_3, \quad g \in \Omega.$$

Then the equality $df^{\#}\omega = f^{\#}d\omega$ holds in the sense of distributions.

References

1. *S. K. Vodopyanov.* Foundations of the Theory of Mappings with Bounded Distortion on Carnot Groups. // Contemporary Mathematics. 2007. V. 424, pp. 303–344.
2. *Yu. G. Reshetnyak.* Space mappings with bounded distortion. Translation of Mathematical Monographs, vol. 73. American Mathematical Society, Providence, RI, 1989.