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V. D. Krevchik, A. V. Razumov,
I. M. Moyko, T.-R. Li, Y.-H. Wang

DOUBLE PHOTO-IONIZATION OF TWO-ELECTRON IMPURITY CENTERS IN QUASI-ZERO-DIMENSIONAL STRUCTURES IN AN EXTERNAL MAGNETIC FIELD

Abstract.

Background. Impurity atoms with two electrons are the simplest systems in which double ionization by a single photon is possible. In this case, influence of an external magnetic field on the double photoionization spectra can lead to new effects associated with the optical absorption dichroism. The aim of this work is to take into account influence of an external magnetic field on the first ionization potential of a two-electron impurity center in a quantum dot, as well as theoretical study of the double photoionization features for two-electron impurity centers in a quasi-zero-dimensional structure in an external magnetic field.

Materials and methods. Influence of an external magnetic field has been taken into account in framework of the perturbation theory. Calculation of the binding energy and the first ionization potential of a two-electron atom has been carried out by a variational method, where the second ionization potential has been taken as an empirical parameter. Expressions for the light impurity absorption coefficients are obtained in the dipole approximation taking into account the quantum dots radius dispersion.

Results. An analytical expression for the first ionization potential of a two-electron impurity center under an external magnetic field is obtained by a variational method within the semi-empirical model framework. Coefficients of the light impurity absorption have been calculated in the dipole approximation, in cases of the light longitudinal and transverse polarization with respect to the magnetic field direction, during photoionization of a two-electron impurity by a single photon.

Conclusions. It is shown that magnetic field has a stabilizing effect on two-electron impurity centers in a semiconductor quantum dot. Dichroism of the optical absorption has been appeared in the absorption band edge shift and in appearance of the additional peaks in the absorption spectral curve in case of the light polarization transverse to the external magnetic field direction ($\vec{e} \perp \vec{B}$) as also in disappearance of the double-hump profile in case of the longitudinal light polarization ($\vec{e} \parallel \vec{B}$).

Key words: two-electron impurity centers, ionization potential, double photoionization, dichroism, quantum dot.

В. Д. Кревчик, А. В. Разумов,
И. М. Мойко, Тян-Ронг Ли, Ю-Хуа Ванг

ДВОЙНАЯ ФОТОИОНИЗАЦИЯ ДВУХЭЛЕКТРОННЫХ ПРИМЕСНЫХ ЦЕНТРОВ В КВАЗИНУЛЬМЕРНЫХ СТРУКТУРАХ ВО ВНЕШНЕМ МАГНИТНОМ ПОЛЕ

Аннотация.

Актуальность и цели. Примесные атомы с двумя электронами представляют собой простейшие системы, в которых возможна двойная ионизация одним фотоном. При этом влияние магнитного поля на спектры двойной фотоионизации может приводить к новым эффектам, связанным с дихроизмом оптического поглощения. Целью данной работы является учет влияния внешнего магнитного поля на первый потенциал ионизации двухэлектронного примесного центра в квантовой точке, а также теоретическое исследование особенностей двойной фотоионизации двухэлектронных примесных центров в квазинульмерной структуре во внешнем магнитном поле.

Материалы и методы. Учет влияния внешнего магнитного поля проводился в рамках теории возмущений. Расчет энергии связи и первого потенциала ионизации двухэлектронного атома осуществлялся вариационным методом, где в качестве эмпирического параметра брался второй потенциал ионизации. Выражения для коэффициентов примесного поглощения света получены в дипольном приближении с учетом дисперсии радиуса квантовых точек.

Результаты. В рамках полуэмпирической модели вариационным методом получено аналитическое выражение для первого потенциала ионизации двухэлектронного примесного центра в условиях внешнего магнитного поля. В дипольном приближении рассчитаны коэффициенты примесного поглощения света в случае продольной и поперечной по отношению к направлению магнитного поля поляризации света, при фотоионизации двухэлектронной примеси одним фотоном.

Выводы. Показано, что магнитное поле оказывает стабилизирующее действие на двухэлектронные примесные центры в полупроводниковой квантовой точке. Дихроизм оптического поглощения проявляется в смещении края полосы поглощения и к появлению на спектральной кривой поглощения дополнительных пиков в случае поперечной по отношению к направлению внешнего магнитного поля ($\vec{e} \perp \vec{B}$) поляризации света и к исчезновению «двугорбого» профиля в случае продольной ($\vec{e} \parallel \vec{B}$) поляризации света.

Ключевые слова: двухэлектронные примесные центры, потенциал ионизации, двойная фотоионизация, дихроизм, квантовая точка.

Introduction

Semiconductor nanostructures are unique objects in which the charge carriers energy spectrum can be controlled, thereby creating device-oriented structures with predetermined characteristics. An important feature of such systems is the strong dependence of their physical properties on the presence of impurities, which inevitably arise in the nanostructures manufacturing process. Two-electron impurity states in quantum dots (QDs) are of special interest. This is due to the fact that, because of the size restriction in all three directions, conditions for the two-electron impurity states existence in a QD are more favorable in comparison with the bulk semiconductor case. In this case, two energy levels, which are formed in the band gap, are divided by the correlation energy $\Delta E = E_2 - E_1$: where E_1 and E_2 are the first and second ionization potentials of the impurity center.

If for the single-electron (hydrogen-like) impurities study in low-dimensional semiconductor systems there is a fairly large number of papers [1, 2], then for the two-electron or helium-like impurities study there are a much less papers. Interest in this type of impurities is associated with possibility for observing one of the fundamental reactions – process of the double photoionization of an impurity atom by a single photon [3]. In addition, it attracts the possibility of controlled modulation of the binding energy for two-electron impurity centers (TIC) and, accordingly, of control for the optical transitions energies by means of an external magnetic field. This is important for practical applications, in particular, in the development of qubits, as well as for photosensors with controlled sensitivity.

Purpose of this work is a variational calculation of the first ionization potential of the quantum dot TIC in an external magnetic field, as well as a theoretical study of the double photoionization features for TIC in a quasi-zero-dimensional structure in an external magnetic field.

1. Variational calculation of the first ionization potential for a two-electron impurity in a semiconductor quantum dot in an external magnetic field

We will calculate the first ionization potential E_1 for TIC with an effective nuclear charge equal to zero ($Z = 0$) in a semiconductive QD in an external magnetic field. Let us use the semiempirical model TIC, developed in [4]. The short-range potential in this model is approximated by a potential well with depth V_0, radius d of which is much smaller than the radius of the localized state. As an empirical parameter, we take the energy of a doubly ionized impurity, i.e. second ionization potential E_2. We will assume that the impurity atom is located at the center of QD. Influence of the magnetic field on the ground state for TIC will be taken into account in framework of the perturbation theory. The two-electron wave function satisfies the Schrödinger equation

$$\mathbf{H}(\rho_1,\rho_2)\Psi(\rho_1,\rho_2) = E\Psi(\rho_1,\rho_2), \quad (1)$$

где

$$\mathbf{H}(\rho_1,\rho_2) = \mathbf{H}(\rho_1) + \mathbf{H}(\rho_2) + \hbar^2 / m^* a_d^2 \left|\vec{\rho}_1 - \vec{\rho}_2\right| + \mathbf{V_B}(\rho_1,\rho_2),$$

$$\mathbf{H}(\rho_i) = -\hbar^2 \Delta_i / 2m^* a_d^2 - \hbar^2 V(\rho_i) / 2m^* a_d^2,$$

$\rho_d = d/a_d$, $V(\rho_i) = \begin{cases} V_0^*, & \rho_i \le \rho_d \\ 0, & \rho_i > \rho_d \end{cases}$, $V_0^* = V_0/E_d$, $\rho_i = r_i/a_d$, r_i – electron coordinates at $i = 1,2$, $a_d = \varepsilon\hbar^2/m^* e^2$, $E_d = \hbar^2/2m^* a_d^2$ – effective Bohr radius and Bohr energy respectively, $\mathbf{V_B}(\rho_1,\rho_2) = m^*\omega_B^2\left(\rho_1^2 + \rho_2^2\right)/4 - i\hbar\omega_B(\partial/\partial\varphi)/2$ – perturbation operator; m^* – effective electron mass, $\omega_B = |e|B/m^*$ – cyclotron frequency.

The value E_1 will be sought by a variational method with wave functions taken as a product of one-electron wave functions $\Psi(\rho i)$:

$$\Psi(\rho_1,\rho_2) = \Psi(\rho_1)\Psi(\rho_2), \quad (2)$$

Using results of [5], where, within framework of the spherically symmetric potential well model (the model of “rigid walls”), an expression for the electron wave function localized at a short-range potential in a QD has been obtained; for a single-electron wave function $\Psi(\rho_i)$, we obtain

$$\Psi(\rho_i)=\frac{B}{\rho_i}\begin{cases}\dfrac{\operatorname{sh}\left(R_0^*\eta^{-1}-\rho_d\eta^{-1}\right)}{\operatorname{sh}\left(R_0^*\eta^{-1}\right)}\dfrac{\sin(\chi_0\rho_i)}{\sin(\chi_0\rho_d)}, \ \rho_i\le\rho_d\\ \dfrac{\operatorname{sh}\left(R_0^*\eta^{-1}-\rho_i\eta^{-1}\right)}{\operatorname{sh}\left(R_0^*\eta^{-1}\right)}, \ \rho_i\ge\rho_d.\end{cases} \quad (3)$$

With account of (3), the trial two-electron wave function is written in the form

$$\Psi(\rho_1,\rho_2)=\frac{B^2}{\rho_1\rho_2}\begin{cases}\dfrac{\operatorname{sh}^2\left(R_0^*\eta^{-1}-\rho_d\eta^{-1}\right)}{\operatorname{sh}^2\left(R_0^*\eta^{-1}\right)}\dfrac{\sin(\chi_0\rho_1)\sin(\chi_0\rho_2)}{\sin^2(\chi_0\rho_d)}, \ \rho_i\le\rho_d\\ \dfrac{\operatorname{sh}\left(R_0^*\eta^{-1}-\rho_1\eta^{-1}\right)\operatorname{sh}\left(R_0^*\eta^{-1}-\rho_2\eta^{-1}\right)}{\operatorname{sh}^2\left(R_0^*\eta^{-1}\right)}, \ \rho_i\ge\rho_d,\end{cases} \quad (4)$$

here $\chi_0=\sqrt{V_0^*-\eta^{-2}}$, $V_0^*=V_0/E_d$, $\eta=\sqrt{E_d/|E_2|}$, $R_0^*=R_0/a_d$, R_0 – the QD radius, $B=\sqrt{2\eta^{-1}\Big/\left(\operatorname{th}\left(R_0^*\eta^{-1}\right)-R_0^*\eta^{-1}\operatorname{cosec}\left(R_0^*\eta^{-1}\right)\right)}$.

In framework of the perturbation theory, the energy of TIC in a magnetic field (in the Bohr units) is determined by

$$\frac{\varepsilon_B\left(R_0^*,\eta\right)}{E_d}=\frac{\varepsilon^{(0)}\left(R_0^*,\eta\right)}{E_d}+\frac{\varepsilon^{(1)}\left(R_0^*,\eta\right)}{E_d}, \quad (5)$$

here $\varepsilon^{(0)}\left(R_0^*,\eta\right)\Big/E_d$ – unperturbed energy of the two-electron impurity center [3]:

$$\frac{\varepsilon^{(0)}\left(R_0^*,\eta\right)}{E_d}=-2^3\eta^{-2}\pi^2\times\operatorname{sh}^{-4}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\left(\operatorname{th}\left(R_0^*\eta^{-1}\right)-R_0^*\eta^{-1}\operatorname{cosec}\left(R_0^*\eta^{-1}\right)\right)^{-2}\Big\{2^{-1}\left(2\eta^{-1}R_0^*-\operatorname{sh}\left(2\eta^{-1}R_0^*\right)\right)^2+$$

$$+\eta\Big[-4\eta^{-1}R_0^*-8\operatorname{ch}^3\left(\eta^{-1}R_0^*\right)\operatorname{sh}\left(\eta^{-1}R_0^*\right)\operatorname{Chi}\left(2\eta^{-1}R_0^*\right)+$$

$$+2\ln\left(2\exp(1+C)\eta^{-1}R_0^*\right)\operatorname{sh}\left(2\eta^{-1}R_0^*\right)+\left(\operatorname{Chi}\left(4\eta^{-1}R_0^*\right)-\ln(2)\right)\operatorname{sh}\left(4\eta^{-1}R_0^*\right)-$$

$$-\operatorname{Shi}\left(2\eta^{-1}R_0^*\right)\left(1-2\operatorname{ch}\left(2\eta^{-1}R_0^*\right)-\operatorname{ch}\left(4\eta^{-1}R_0^*\right)\right)+\operatorname{ch}\left(4\eta^{-1}R_0^*\right)\operatorname{Shi}\left(4\eta^{-1}R_0^*\right)\Big]\Big\}, \quad (6)$$

где $\mathrm{Chi}(x)$ и $\mathrm{Shi}(x)$ – the integral hyperbolic cosine and sine respectively, $C = 0{,}577$ – the Euler's constant.

According to the perturbation theory, the first approximation correction $\varepsilon^{(1)}\left(R_0^*,\eta\right)/E_d$ will be determined by the average value of perturbation potential in the state $\Psi^{(0)}(\rho_1,\rho_2)$:

$$\frac{\varepsilon^{(1)}\left(R_0^*,\eta\right)}{E_d} = \left\langle \Psi^{(0)}(\rho_1,\rho_2) \middle| \mathbf{V}_B(\rho_1,\rho_2) \middle| \Psi^{(0)}(\rho_1,\rho_2) \right\rangle. \tag{7}$$

With the expression account for the unperturbed two-electron wave function (4), for (7) we have

$$\frac{\varepsilon^{(1)}\left(R_0^*,\eta\right)}{E_d} = -\frac{\pi^2 2^2 m^* \omega_B^2 a_d^2 \eta^{-2}}{sh^4\left(R_0^*\eta^{-1}\right)\left(th\left(R_0^*\eta^{-1}\right) - R_0^*\eta^{-1}\cos ec\left(R_0^*\eta^{-1}\right)\right)^2} \times$$

$$\times \int_0^{\pi} d\theta_1 \sin\theta_1 \int_0^{2\pi} d\varphi_1 \int_0^{\pi} d\theta_2 \sin\theta_2 \int_0^{2\pi} d\varphi_2 \left[\frac{sh^4\left(R_0^*\eta^{-1} - \rho_d\eta^{-1}\right)}{\sin^4(\chi_0\rho_d)} \theta(\rho_d - \rho_1) \times \right.$$

$$\times \theta(\rho_d - \rho_2) \int_0^{\rho_d} d\rho_1 \int_0^{\rho_d} d\rho_2 \left(\rho_1^2 + \rho_2^2\right) \sin^2(\chi_0\rho_1) \sin^2(\chi_0\rho_2) + \theta(\rho_1 - \rho_d) \times$$

$$\left. \times \theta(\rho_2 - \rho_d) \int_{\rho_d}^{R_0^*} d\rho_1 \int_{\rho_d}^{R_0^*} d\rho_2 \left(\rho_1^2 + \rho_2^2\right) sh^2\left(R_0^*\tilde{\eta}^{-1} - \rho_1\tilde{\eta}^{-1}\right) sh^2\left(R_0^*\tilde{\eta}^{-1} - \rho_2\tilde{\eta}^{-1}\right) \right]. \tag{8}$$

The integrals in (8) are as follows:

$$\int_0^{\rho_d} d\rho_1 \int_0^{\rho_d} d\rho_2 \left(\rho_1^2 + \rho_2^2\right) \sin^2(\chi_0\rho_1) \sin^2(\chi_0\rho_2) = 2^{-5} 3^{-1} \chi_0^{-4} \times$$

$$\times \left[-3 + 6\chi_0^2\rho_d^2 + 16\chi_0^4\rho_d^4 - 24\chi_0^2\rho_d^2 \cos(2\chi_0\rho_d) + \left(3 - 6\chi_0^2\rho_d^2\right)\cos(4\chi_0\rho_d) + \right.$$

$$\left. + 12\chi_0\rho_d \sin(2\chi_0\rho_d) - 32\chi_0^3\rho_d^3 \sin(2\chi_0\rho_d) + 6\chi_0\rho_d \sin(4\chi_0\rho_d) \right], \tag{9}$$

$$\int_{\rho_d}^{R_0^*} d\rho_1 \int_{\rho_d}^{R_0^*} d\rho_2 \left(\rho_1^2 + \rho_2^2\right) \mathrm{sh}^2\left(R_0^*\eta^{-1} - \rho_1\eta^{-1}\right) \mathrm{sh}^2\left(R_0^*\eta^{-1} - \rho_2\eta^{-1}\right) = 2^{-5} 3^{-1} \eta^4 \times$$

$$\times \left[-3 + 24R_0^{*2}\eta^{-2} + 16R_0^{*4}\eta^{-4} - 24R_0^*\eta^{-2}\rho_d - 16R_0^{*3}\eta^{-4}\rho_d - 6\eta^{-2}\rho_d^2 - \right.$$

$$-16R_0^*\eta^{-4}\rho_d^3+16\eta^{-4}\rho_d^4-24\eta^{-2}\left(R_0^*-\rho_d\right)\rho_d\mathrm{ch}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)+$$

$$+\left(3+6\eta^{-2}\rho_d^2\right)\mathrm{ch}\left(4\eta^{-1}\left(R_0^*-\rho_d\right)\right)-24\eta^{-1}R_0^*\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)-$$

$$-8\eta^{-3}R_0^{*3}\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)+12\eta^{-1}\rho_d\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)-$$

$$-24\eta^{-3}R_0^*\rho_d^2\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)+32\eta^{-3}\rho_d^3\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)+$$

$$+6\eta^{-1}\rho_d\mathrm{sh}\left(4\eta^{-1}\left(R_0^*-\rho_d\right)\right)\Big]. \qquad (10)$$

Then expression for the correction to energy (7) is written as

$$\frac{\varepsilon^{(1)}\left(R_0^*,\eta\right)}{E_d}=\frac{m^*\omega_B^2a_d^2\pi^2\eta^{-2}}{6sh^4\left(R_0^*\eta^{-1}\right)\left(\mathrm{th}\left(R_0^*\eta^{-1}\right)-R_0^*\eta^{-1}\mathrm{csch}\left(R_0^*\eta^{-1}\right)\right)^2}\times$$

$$\times\left\{\frac{\mathrm{sh}^4\left(R_0^*\eta^{-1}-\rho_d\eta^{-1}\right)}{\chi_0^4\sin^4(\chi_0\rho_d)}\times\Big[-3+6\chi_0^2\rho_d^2+16\chi_0^4\rho_d^4-24\chi_0^2\rho_d^2\cos(2\chi_0\rho_d)+\right.$$

$$+\left(3-6\chi_0^2\rho_d^2\right)\cos(4\chi_0\rho_d)+12\chi_0\rho_d\sin(2\chi_0\rho_d)-32\chi_0^3\rho_d^3\sin(2\chi_0\rho_d)+$$

$$+6\chi_0\rho_d\sin(4\chi_0\rho_d)\Big]+\eta^4\Big[-3+24R_0^{*2}\eta^{-2}+16R_0^{*4}\eta^{-4}-24R_0^*\eta^{-2}\rho_d-$$

$$-16R_0^{*3}\eta^{-4}\rho_d-6\eta^{-2}\rho_d^2-16R_0^*\eta^{-4}\rho_d^3+16\eta^{-4}\rho_d^4-24\eta^{-2}\left(R_0^*-\rho_d\right)\rho_d\times$$

$$\times\mathrm{ch}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)+\left(3+6\eta^{-2}\rho_d^2\right)\mathrm{ch}\left(4\eta^{-1}\left(R_0^*-\rho_d\right)\right)+$$

$$-24\eta^{-1}R_0^*\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)-8\eta^{-3}R_0^{*3}\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)+$$

$$+12\eta^{-1}\rho_d\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)-24\eta^{-3}R_0^*\rho_d^2\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)+$$

$$\left.+32\eta^{-3}\rho_d^3\mathrm{sh}\left(2\eta^{-1}\left(R_0^*-\rho_d\right)\right)+6\eta^{-1}\rho_d\mathrm{sh}\left(4\eta^{-1}\left(R_0^*-\rho_d\right)\right)\Big]\right\}. \qquad (11)$$

By making in (11) the limiting transition ($d\to 0$), for $\varepsilon^{(1)}\left(R_0^*,\eta\right)/E_d$ we obtain

$$\frac{\varepsilon^{(1)}\left(R_0^*,\eta\right)}{E_d}=-\frac{m^*\omega_B^2\pi^2\eta^{-2}}{6\mathrm{sh}^4E_d\left(R_0^*\eta^{-1}\right)\left(\mathrm{th}\left(R_0^*\eta^{-1}\right)-R_0^*\eta^{-1}\mathrm{csch}\left(R_0^*\eta^{-1}\right)\right)^2}\times$$

$$\times\left[-3+24R_0^{*2}\eta^{-2}+16R_0^{*4}\eta^{-4}+3\mathrm{ch}\left(4\eta^{-1}R_0^*\right)-\right.$$

$$\left.-8\eta^{-1}R_0^*\left(3+\eta^{-2}R_0^{*2}\right)\mathrm{sh}\left(2\eta^{-1}R_0^*\right)\right], \qquad (12)$$

Taking into account (6) for the TIC energy (5) we have

$$\frac{\varepsilon_B\left(R_0^*,\eta\right)}{E_d}=-2^3\eta^{-2}\pi^2\times\mathrm{sh}^{-4}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\left(\mathrm{th}\left(R_0^*\eta^{-1}\right)-R_0^*\eta^{-1}\mathrm{cosec}\left(R_0^*\eta^{-1}\right)\right)^{-2}\left\{2^{-1}\left(2\eta^{-1}R_0^*-\mathrm{sh}\left(2\eta^{-1}R_0^*\right)\right)^2+\right.$$

$$+\eta\left[-4\eta^{-1}R_0^*-8\,\mathrm{ch}^3\left(\eta^{-1}R_0^*\right)\mathrm{sh}\left(\eta^{-1}R_0^*\right)\mathrm{Chi}\left(2\eta^{-1}R_0^*\right)+\right.$$

$$+2\ln\left(2\exp(1+C)\eta^{-1}R_0^*\right)\mathrm{sh}\left(2\eta^{-1}R_0^*\right)+\left(\mathrm{Chi}\left(4\eta^{-1}R_0^*\right)-\ln(2)\right)\mathrm{sh}\left(4\eta^{-1}R_0^*\right)-$$

$$\left.\left.-\mathrm{Shi}\left(2\eta^{-1}R_0^*\right)\left(1-2\mathrm{ch}\left(2\eta^{-1}R_0^*\right)-\mathrm{ch}\left(4\eta^{-1}R_0^*\right)\right)+\mathrm{ch}\left(4\eta^{-1}R_0^*\right)\mathrm{Shi}\left(4\eta^{-1}R_0^*\right)\right]\right\}-$$

$$-\frac{m^*\omega_B^2\pi^2\eta^{-2}}{6sh^4E_d\left(R_0^*\eta^{-1}\right)\left(th\left(R_0^*\eta^{-1}\right)-R_0^*\eta^{-1}\mathrm{csch}\left(R_0^*\eta^{-1}\right)\right)^2}\times\left[-3+24R_0^{*2}\eta^{-2}+\right.$$

$$\left.+16R_0^{*4}\eta^{-4}+3\mathrm{ch}\left(4\eta^{-1}R_0^*\right)-8\eta^{-1}R_0^*\left(3+\eta^{-2}R_0^{*2}\right)\mathrm{sh}\left(2\eta^{-1}R_0^*\right)\right]. \qquad (13)$$

Minimization of the expression (13) requires to find an extreme value $\tilde{\eta}$, which requires to solve the transcendental equation

$$\frac{\partial\varepsilon_B\left(R_0^*,\eta\right)}{\partial\eta}=0. \qquad (14)$$

The explicit expression for (14) is:

$$\left\{\frac{2^7\pi^2\tilde{\eta}^{-2}}{\mathrm{sh}^4\left(R_0^*\tilde{\eta}^{-1}\right)\left(\mathrm{th}\left(R_0^*\tilde{\eta}^{-1}\right)-R_0^*\tilde{\eta}^{-1}\mathrm{cosec}\left(R_0^*\tilde{\eta}^{-1}\right)\right)^3}\times\right.$$

$$\times\left[R_0^*\eta^{-2}\csc\left(2\eta^{-1}R_0^*\right)-R_0^*\tilde{\eta}^{-3}\mathrm{ctg}\left(2\tilde{\eta}^{-1}R_0^*\right)\csc\left(2\tilde{\eta}^{-1}R_0^*\right)-R_0^*\tilde{\eta}^{-2}\mathrm{sech}\left(2\tilde{\eta}^{-1}R_0^*\right)\right]+$$

$$\left.+\frac{2^7\pi^2\tilde{\eta}^{-3}-2^8\pi^2\tilde{\eta}^{-4}\mathrm{cth}\left(R_0^*\tilde{\eta}^{-1}\right)}{\mathrm{sh}^4\left(R_0^*\tilde{\eta}^{-1}\right)\left(\mathrm{th}\left(R_0^*\tilde{\eta}^{-1}\right)-R_0^*\tilde{\eta}^{-1}\mathrm{cosec}\left(R_0^*\tilde{\eta}^{-1}\right)\right)^2}\right\}\left\{\frac{1}{16}\left(2R_0^*\tilde{\eta}^{-1}-\mathrm{sh}\left(2\tilde{\eta}^{-1}R_0^*\right)\right)^2+\right.$$

$$+\frac{1}{8}\tilde{\eta}\Big[-4R_0^*\tilde{\eta}^{-1}-8\mathrm{ch}^3\left(2\tilde{\eta}^{-1}R_0^*\right)\mathrm{Ci}\left(2\tilde{\eta}^{-1}R_0^*\right)\mathrm{sh}\left(\tilde{\eta}^{-1}R_0^*\right)+$$

$$+2\ln\left(\tilde{\eta}^{-1}R_0^*e^{1+\gamma}\right)\mathrm{sh}\left(2\tilde{\eta}^{-1}R_0^*\right)+\left(\mathrm{Ci}\left(4\tilde{\eta}^{-1}R_0^*\right)-\ln 2\right)\mathrm{sh}\left(4\tilde{\eta}^{-1}R_0^*\right)-\mathrm{Si}\left(2\tilde{\eta}^{-1}R_0^*\right)\times$$

$$\times\left(1-2\mathrm{ch}\left(2\tilde{\eta}^{-1}R_0^*\right)-\mathrm{ch}\left(4\tilde{\eta}^{-1}R_0^*\right)\right)-\mathrm{Si}\left(4\tilde{\eta}^{-1}R_0^*\right)+\mathrm{ch}\left(4\tilde{\eta}^{-1}R_0^*\right)\mathrm{Si}\left(4\tilde{\eta}^{-1}R_0^*\right)\Big]\Big]\Big\}-$$

$$-\frac{2^6\pi^2\tilde{\eta}^{-2}}{\mathrm{sh}^4\left(R_0^*\tilde{\eta}^{-1}\right)\left(\mathrm{th}\left(R_0^*\tilde{\eta}^{-1}\right)-R_0^*\tilde{\eta}^{-1}\mathrm{cosec}\left(R_0^*\tilde{\eta}^{-1}\right)\right)^2}\times$$

$$\times\Big\{\Big[\frac{1}{8}\left(2R_0^*\tilde{\eta}^{-2}-2\tilde{\eta}^{-2}\mathrm{ch}\left(2\tilde{\eta}^{-1}R_0^*\right)\right)^2\left(2R_0^*\tilde{\eta}^{-1}-\mathrm{sh}\left(2\tilde{\eta}^{-1}R_0^*\right)\right)+$$

$$+\frac{\tilde{\eta}}{8}\Big[4R_0^*\tilde{\eta}^{-2}+8\tilde{\eta}^{-2}R_0^*\mathrm{ch}^4\left(\tilde{\eta}^{-1}R_0^*\right)\mathrm{Ci}\left(2\tilde{\eta}^{-1}R_0^*\right)+4\tilde{\eta}^{-2}R_0^*\mathrm{ch}^4\left(4\tilde{\eta}^{-1}R_0^*\right)\times$$

$$+2\ln\left(\tilde{\eta}^{-1}R_0^*e^{1+\gamma}\right)\mathrm{sh}\left(2\tilde{\eta}^{-1}R_0^*\right)+\left(\mathrm{Ci}\left(4\tilde{\eta}^{-1}R_0^*\right)-\ln 2\right)\mathrm{sh}\left(4\tilde{\eta}^{-1}R_0^*\right)-\mathrm{Si}\left(2\tilde{\eta}^{-1}R_0^*\right)\times$$

$$\times\left(1-2\mathrm{ch}\left(2\tilde{\eta}^{-1}R_0^*\right)-\mathrm{ch}\left(4\tilde{\eta}^{-1}R_0^*\right)\right)-\mathrm{Si}\left(4\tilde{\eta}^{-1}R_0^*\right)+\mathrm{ch}\left(4\tilde{\eta}^{-1}R_0^*\right)\mathrm{Si}\left(4\tilde{\eta}^{-1}R_0^*\right)\Big]\Big]+$$

$$+\frac{1}{8}\Big[-4R_0^*\tilde{\eta}^{-1}-8\mathrm{ch}^3\left(2\tilde{\eta}^{-1}R_0^*\right)\mathrm{Ci}\left(2\tilde{\eta}^{-1}R_0^*\right)\mathrm{sh}\left(\tilde{\eta}^{-1}R_0^*\right)+$$

$$\times\left(\mathrm{Ci}\left(2\tilde{\eta}^{-1}R_0^*\right)-\ln 2\right)-4\tilde{\eta}^{-2}R_0^*\ln\left(\tilde{\eta}^{-1}R_0^*e^{1+\gamma}\right)\mathrm{ch}\left(2\tilde{\eta}^{-1}R_0^*\right)+8\tilde{\eta}^{-1}\mathrm{ch}^3\left(\tilde{\eta}^{-1}R_0^*\right)\times$$

$$\times\mathrm{ch}\left(2\tilde{\eta}^{-1}R_0^*\right)\mathrm{sh}\left(\tilde{\eta}^{-1}R_0^*\right)+24\tilde{\eta}^{-1}R_0^*\mathrm{ch}^2\left(\tilde{\eta}^{-1}R_0^*\right)\mathrm{Ci}\left(2\tilde{\eta}^{-1}R_0^*\right)\mathrm{sh}^2\left(\tilde{\eta}^{-1}R_0^*\right)-$$

$$-2\tilde{\eta}^{-1}\mathrm{sh}\left(2\tilde{\eta}^{-1}R_0^*\right)-\tilde{\eta}^{-1}\mathrm{sh}\left(2\tilde{\eta}^{-1}R_0^*\right)\left(1-2\mathrm{ch}\left(2\tilde{\eta}^{-1}R_0^*\right)-\mathrm{ch}\left(4\tilde{\eta}^{-1}R_0^*\right)\right)+$$

$$+\tilde{\eta}^{-1}\mathrm{sh}\left(4\tilde{\eta}^{-1}R_0^*\right)-2\tilde{\eta}^{-1}\mathrm{ch}\left(4\tilde{\eta}^{-1}R_0^*\right)\mathrm{sh}\left(4\tilde{\eta}^{-1}R_0^*\right)-4\tilde{\eta}^{-1}R_0^*\mathrm{Si}\left(2\tilde{\eta}^{-1}R_0^*\right)\times$$

$$\times\left(\mathrm{sh}\left(4\tilde{\eta}^{-1}R_0^*\right)+\mathrm{sh}\left(4\tilde{\eta}^{-1}R_0^*\right)\right)-4\tilde{\eta}^{-1}R_0^*\mathrm{sh}\left(4\tilde{\eta}^{-1}R_0^*\right)\mathrm{Si}\left(4\tilde{\eta}^{-1}R_0^*\right)\Big\}+$$

$$+\frac{m^*\omega_B^2a_d^2\pi^2\tilde{\eta}^{-2}}{\mathrm{sh}^4\left(R_0^*\tilde{\eta}^{-1}\right)\left(\mathrm{th}\left(R_0^*\tilde{\eta}^{-1}\right)-R_0^*\tilde{\eta}^{-1}\mathrm{cosec}\left(R_0^*\tilde{\eta}^{-1}\right)\right)^2}\times$$

$$\times\Big\{\Big[-R_0^*\Big[\left(R_0^*\mathrm{ctg}\left(R_0^*\tilde{\eta}^{-1}\right)-\tilde{\eta}\right)\mathrm{cosec}\left(R_0^*\tilde{\eta}^{-1}\right)+\tilde{\eta}\,\mathrm{sech}^2\left(R_0^*\tilde{\eta}^{-1}\right)\Big]+$$

$$+\left(2R_0^*\mathrm{ctgh}\left(R_0^*\tilde{\eta}\right)-\tilde{\eta}\right)\left(R_0^*\mathrm{cosec}\left(R_0^*\tilde{\eta}^{-1}\right)-\tilde{\eta}\,\mathrm{tgh}\left(R_0^*\tilde{\eta}^{-1}\right)\right)\Big]\times$$

$$\times\left[16R_0^{*4}+24R_0^{*2}\tilde{\eta}^2-3\tilde{\eta}^4\left(1-\mathrm{ch}\left(4R_0^*\tilde{\eta}^{-1}\right)\right)-8R_0^*\tilde{\eta}\left(R_0^{*2}+3\tilde{\eta}^2\right)\mathrm{sh}\left(2R_0^*\tilde{\eta}\right)\right]+$$

$$+2R_0^*\tilde{\eta}\left[-16R_0^{*3}-12R_0^*\tilde{\eta}^2+4R_0^*\left(R_0^{*2}+3\tilde{\eta}^2\right)\mathrm{csch}\left(2R_0^*\tilde{\eta}^{-1}\right)+6\tilde{\eta}\left(R_0^{*2}+\tilde{\eta}^2\right)\times\right.$$

$$\left.\times\mathrm{sh}\left(2R_0^*\tilde{\eta}^{-1}\right)-3\tilde{\eta}^3\,\mathrm{sh}\left(4R_0^*\tilde{\eta}^{-1}\right)\right]\left(R_0^*\csc\left(R_0^*\tilde{\eta}^{-1}\right)-\tilde{\eta}\,\mathrm{tgh}\left(R_0^*\tilde{\eta}^{-1}\right)\right)\Big]\Big\}=0\,. \quad (15)$$

If we take into account, that the minimum value of the functional $\varepsilon\left(R_0^*,\eta\right)/E_d$, achieved with an extreme value of the parameter $\eta=\tilde{\eta}$, is the sum

$$-\frac{\varepsilon_B\left(R_0^*,\tilde{\eta}\right)}{E_d}=\frac{E_1}{E_d}+\frac{E_2}{E_d}, \quad (16)$$

we can find the first potential as a function of the second, taken from the experiment

$$\frac{E_1}{E_d}=-\frac{\varepsilon_B\left(R_0^*,\tilde{\eta}\right)}{E_d}-\frac{E_2}{E_d}. \quad (17)$$

Figure 1 shows the relationship between the first and second ionization potentials for TIC with the zero nuclear charge ($Z = 0$) in semiconductive QD, taking into account the influence of an external magnetic field, in the Bohr energy E_d units, obtained from (17) by numerical calculations. Curves 1, 2, 3 correspond to different values of the magnetic field induction B. Comparison of curves 1, 2 and 3 shows that in a magnetic field the threshold value of the second ionization potential for a two-electron impurity decreases, and thus the magnetic field has a stabilizing effect on the TIC in a semiconductive QD, which is caused by the Landau levels dynamics in an external magnetic field and by suppression of the Coulomb electron interactions.

2. The absorption dichroism under double photoionization of two-electron impurity centers for a quasi-zero-dimensional structure in an external magnetic field

Let us consider the photoionization process for TIC in an external magnetic field in a semiconductive QD. Wave function of the initial state is determined by the expression

$$\Psi\left(\rho_1,\rho_2\right)=\frac{2^2\eta^{-2}}{\left(\mathrm{th}\left(R_0^*\eta^{-1}\right)-R_0^*\eta^{-1}\mathrm{cosec}\left(R_0^*\eta^{-1}\right)\right)^2}\times$$

$$\times\frac{\mathrm{sh}\left(R_0^*\eta^{-1}-\rho_1\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}-\rho_2\eta^{-1}\right)}{\rho_1\rho_2\mathrm{sh}^2\left(R_0^*\eta^{-1}\right)}. \quad (18)$$

Wave function of the final state is taken as a product of the wave functions for electrons in a spherical QD

$$\Phi(\rho_1,\rho_2) = \Psi_{n,l,m}(\rho_1,\varphi_1,\theta_1)\Psi_{n,l,m}(\rho_2,\varphi_2,\theta_2)\,. \qquad (19)$$

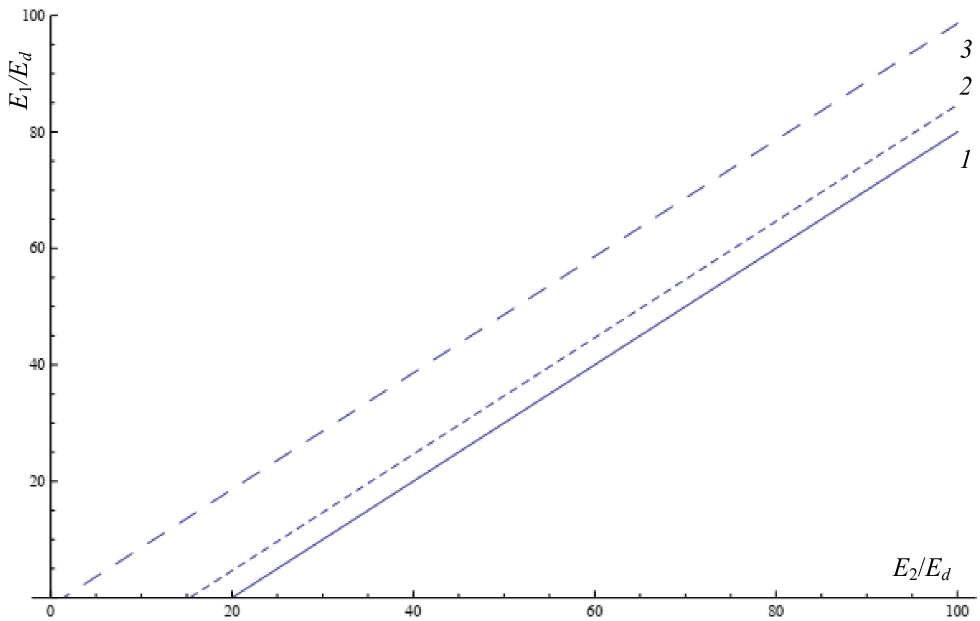

Fig. 1. Relationship between the first E_1 and second E_2 ionization potentials of a two-electron impurity center with the zero nuclear charge (Z = 0) in a semiconductive QD for different values of an external magnetic field induction *B*: *1* – $B = 0$; *2* – $B = 2$ T; *3* – $B = 3$ T

Influence of the magnetic field on the electron final states in a QD will be taken into account in framework of the perturbation theory. Then, in the second order of the perturbation theory, the electron wave function will have the form [6]:

$$\Psi_{n,l,m}(r,\theta,\varphi) = \Psi^{(0)}_{n,l,m}(r,\theta,\varphi) + \sum_{n'l'm'} \frac{R_0^{*2} V_{n,l,m;n',l',m'}}{\pi^2 - \tilde{X}^2_{n',l'}} \Psi^{(0)}_{n',l',m'}(r,\theta,\varphi), \quad (20)$$

where $\Psi^{(0)}_{n,l,m}(r,\theta,\varphi)$ – the zero-order wave function:

$$\Psi^{(0)}_{n,l,m}(r,\theta,\varphi) = Y_{l,m}(\theta,\varphi) \frac{J_{l+\frac{3}{2}}\left(\frac{\tilde{X}_{n,l}}{R_0^*} r^*\right)}{a_h^{3/2}\sqrt{2\pi}R_0^*\sqrt{r^*}\,J_{l+\frac{3}{2}}\left(\tilde{X}_{n,l}\right)}, \qquad (21)$$

here $i = 1, 2$; $Y_{l,m}$ – the normalized ball functions; l, m – the orbital and magnetic quantum numbers; $J_\nu(x)$ – the first order Bessel function with form ν; $k_{nl} = \xi_{nl}/R_0$; ξ_{nl} – n-th root for Bessel function of the l-th order.

In the second order of perturbation theory, the electron energy spectrum in an external magnetic field can be written as [6]:

$$E = E^{(0)} + V_{n,l,m;n,l,m} + \sum_{n'l'm'} \frac{R_0^{*2} \left| V_{n,l,m;n',l',m'} \right|^2}{\pi^2 - \tilde{X}_{n',l'}^2}, \tag{22}$$

here $\tilde{X}_{n',l'}$ – root of the Bessel function with half-integer order $l+1/2$, $E^{(0)} = \tilde{X}_{n,l}^2 E_h \big/ R_0^{*2}$ – zero approximation to electron energy in a dimensionally quantized band, $V_{n,l,m;n',l',m'}$ – matrix element of the perturbation operator, which in spherical coordinates has the next view

$$V_{n,l,m;n'l'm'} = \sum_{n'=0}^{\infty} \left\{ \frac{\hbar\omega_B m}{4\pi\left(\tilde{X}_{n,l}^2 - \tilde{X}_{n',l}^2\right) J_{l+\frac{3}{2}}\left(\tilde{X}_{n,l}\right) J_{l+\frac{3}{2}}\left(\tilde{X}_{n',l}\right)} \times \right.$$

$$\times \left[R_0^* \tilde{X}_{n',l} J_{l+\frac{1}{2}}\left(R_0^* \tilde{X}_{n',l}\right) J_{l+\frac{3}{2}}\left(R_0^* \tilde{X}_{n,l}\right) - \right.$$

$$\left. - R_0^* \tilde{X}_{n,l} J_{l+\frac{1}{2}}\left(R_0^* \tilde{X}_{n,l}\right) J_{l+\frac{3}{2}}\left(R_0^* \tilde{X}_{n',l}\right) \right] + \sum_{k=0}^{\infty} \frac{(-1)^k m_h^* \omega_B^2 R_0^* \left(\tilde{X}_{n,l}^{l+\frac{3}{2}}\right)^{2k+1}}{k!\,\Gamma\left(l+k+\frac{5}{2}\right) 2^{l+4+2k}} \times$$

$$\times \left[\sqrt{\frac{(l-m+4)(l-m+3)(l-m+2)(l-m+1)}{(2l+1)(2l-1)^2(2l-3)}} \frac{F\left(-k, -l-k-\frac{3}{2}, l+\frac{5}{2}, \frac{\tilde{X}_{n',l-2}^2}{\tilde{X}_{n,l}^2}\right)}{(2l+2k+2)\Gamma\left(l+\frac{1}{2}\right)} - \right.$$

$$- \frac{2F\left(-k, -l-k-\frac{3}{2}, l+\frac{5}{2}, \frac{\tilde{X}_{n',l}^2}{\tilde{X}_{n,l}^2}\right)}{(2l+3)(2l-1)(2l+2k+4)} \sqrt{(l-m)(l-m-1)(l+m+2)(l+m+1)} +$$

$$+ \frac{F\left(-k, -l-k-\frac{3}{2}, l+\frac{5}{2}, \frac{\tilde{X}_{n',l+2}^2}{\tilde{X}_{n,l}^2}\right)}{2l+2k+6} \times$$

$$\left. \left. \times \sqrt{\frac{(l+m+4)(l+m+3)(l+m+2)(l+m+1)}{(2l+5)(2l+3)^2(2l+1)}} \right] \right\}. \tag{23}$$

The effective Hamiltonian for interaction with the light wave field in case of the longitudinal polarization $\mathbf{e}_\lambda$ with respect to the magnetic field direction ($\vec{e}_\lambda \perp \vec{B}$) is determined by the next expression:

$$\mathbf{H}_{\mathrm{int}\,B}^{(s)} = -i\lambda_0\hbar\left(\frac{2\pi\hbar^2\alpha^*}{\varepsilon\omega m^{*2}} I_0\right)^{1/2} \exp(i\mathbf{q}_s\mathbf{r})(\mathbf{e}_{\lambda s}\nabla_{\mathbf{r}}), \quad (24)$$

where λ_0 – coefficient of the local field, taking into account the amplitudes difference for the local and average macroscopic fields; I_0 – the light intensity; ω – the absorbed light frequency; ε – the static dielectric permeability of the QD material; α^* - the fine structure constant with account of dielectric permeability.

The matrix element, which determines magnitude of the oscillator strength for the dipole optical transitions of electrons from the ground state of TIC (18) to the discrete spectrum state $\Psi_{n,m,l}(\rho,\varphi,\theta)$ of QD (20), in case of the light polarization along the magnetic field direction, will be written as

$$M^{(s)} = i\lambda_0\sqrt{\frac{2\pi\alpha^*}{\omega} I_0}\Big[\left(E_{n,l,m} - E_1\right)\times$$

$$\times\left\langle \Psi_{n,l,m}^*(\rho_1,\theta_1,\varphi_1)\psi^*(\rho_2)\right|(\mathbf{e}_\lambda,\mathbf{r}_1)\left|\Psi(\rho_1,\rho_2)\right\rangle +$$

$$+\left(E_{n,l,m} - E_2\right)\left\langle \Psi_{n,l,m}^*(\rho_1,\theta_1,\varphi_1)\psi^*(\rho_2)\right|(\mathbf{e}_\lambda,\mathbf{r}_2)\left|\Psi(\rho_1,\rho_2)\right\rangle\Big]. \quad (25)$$

Calculation of the matrix element (25) leads to the following integrals

$$\int_0^{2\pi} \exp(-im\varphi)\,d\varphi = \begin{cases} 0, \text{ if } m \neq 0, \\ 2\pi, \text{ if } m = 0. \end{cases} \quad (26)$$

As it can be seen from (26), that the selection rule for the magnetic quantum number m is such that in case of $\vec{e}_\lambda \parallel \vec{B}$ optical transitions from the impurity level are possible only in the QD states with value $m = 0$.

Taking into account (18)–(26), expression for the square modulus of the matrix element (25) can be written as

$$\left|M^{(s)}\right|^2 = \frac{\alpha^*}{\hbar\omega}\frac{\pi\lambda_0^2 I_0\hbar E_d^2}{a_d^2 R_0^{*2}\mathrm{sh}^6\left(R_0^*\eta^{-1}\right)}\times\frac{2^5\eta^{-3}}{\left(\mathrm{th}\left(R_0^*\eta^{-1}\right) - R_0^*\eta^{-1}\mathrm{cosec}\left(R_0^*\eta^{-1}\right)\right)^3}\times$$

$$\times\left(2\xi_{nl}^2 R_0^{*-2} + V_{n,l,0;n,l,m} + \sum_{n'l'm'}\frac{R_0^{*2}\left|V_{n,l,0;n',l',m'}\right|^2}{\pi^2 - \tilde{X}_{n',l'}^2} + |E_1|/E_d + |E_2|/E_d\right)^2\times$$

$$\times\left|\frac{\sqrt{k_{n1}^{-3}\left(k_{n1}^{2}+\eta^{-2}\right)^{-2}}\sqrt{k_{n1}+i\eta^{-1}}\left(k_{n1}-i2\eta^{-1}\right)}{J_{l+3/2}\left(\xi_{nl}\right)}\times\right.$$

$$\times\mathrm{ch}\left(R_0^*\eta^{-1}\right)S\left(\sqrt{\frac{2}{\pi}}R_0^*\left(k_{n1}-i\eta^{-1}\right)\right)+i\sqrt{k_{n1}-i\eta^{-1}}\left(k_{n1}+i2\eta^{-1}\right)\mathrm{ch}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Si}\left(\sqrt{\frac{2}{\pi}}R_0^*\left(k_{n1}+i\eta^{-1}\right)\right)+\sqrt{k_{n1}+i\eta^{-1}}\left(k_{n1}-i2\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2}{\pi}}R_0^*\left(k_{n1}-i\eta^{-1}\right)\right)+\sqrt{k_{n1}-i\eta^{-1}}\left(k_{n1}+i2\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2}{\pi}}R_0^*\left(k_{n1}+i\eta^{-1}\right)\right)\Bigg]\sqrt{\frac{2}{\pi k_{n1}}}\left(\eta^{-2}+k_{n1}^2\right)^{-2}\times$$

$$\times\left\{\eta^{-1}\left[\left(\eta^{-2}+k_{n1}^2\right)R_0^*\cos\left(k_{n1}R_0^*\right)-\left(\eta^{-2}+2k_{n1}+k_{n1}^2\right)R_0^*\sin\left(k_{n1}R_0^*\right)\right]+\right.$$

$$\left.+\left(\eta^{-2}\left(k_{nl}-1\right)+k_{nl}^2\left(k_{nl}+1\right)\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\right\}+$$

$$+\sum_{n'l'm'}\frac{R_0^{*2}V_{n,l,0;n',l',m'}}{\pi^2-\tilde{X}_{n',l'}^2}\frac{\sqrt{k_{n'1}^{-3}\left(k_{n'1}^{2}+\eta^{-2}\right)^{-2}}\sqrt{k_{n'1}+i\eta^{-1}}\left(k_{n'1}-i2\eta^{-1}\right)}{J_{l'+3/2}\left(\xi_{n'l}\right)}\times$$

$$\times\mathrm{ch}\left(R_0^*\eta^{-1}\right)\mathrm{Si}\left(\sqrt{\frac{2R_0^*}{\pi}}\left(k_{n'1}-i\eta^{-1}\right)\right)+i\sqrt{k_{n'1}-i\eta^{-1}}\left(k_{n'1}+i2\eta^{-1}\right)\mathrm{ch}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Si}\left(\sqrt{\frac{2R_0^*}{\pi}}\left(k_{n'1}+i\eta^{-1}\right)\right)+\sqrt{k_{n'1}+i\eta^{-1}}\left(k_{n'1}-i2\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2R_0^*}{\pi}}\left(k_{n'1}-i\eta^{-1}\right)\right)+\sqrt{k_{n'1}-i\eta^{-1}}\left(k_{n'1}+i2\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2R_0^*}{\pi}}\left(k_{n'1}+i\eta^{-1}\right)\right)\Bigg]\sqrt{\frac{2}{\pi k_{n'1}}}\left(\eta^{-2}+k_{n'1}^2\right)^{-2}\times$$

$$\times\left\{\eta^{-1}\left[\left(\eta^{-2}+k_{n'1}^2\right)R_0^*\cos\left(k_{n'1}R_0^*\right)-\left(\eta^{-2}+2k_{n'1}+k_{n''1}^2\right)R_0^*\sin\left(k_{n'1}R_0^*\right)\right]+\right.$$

$$+\left(\eta^{-2}\left(k_{n'l'}-1\right)+k_{n'l'}^{2}\left(k_{n'l'}+1\right)\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\Big\}\Bigg|^2, \qquad (27)$$

here $\mathrm{Ci}(x)$ и $\mathrm{Si}(x)$ – the integral cosine and sine respectively.

We assume that the sizes dispersion u for QDs arises in process of the phase decomposition of a supersaturated solid solution and is satisfactorily described by the Lifshitz-Slezov formula [7]:

$$P(u)=\begin{cases}\dfrac{3^4 e u^2 \exp\left[-1/\left(1-2u/3\right)\right]}{2^{\frac{5}{3}}\left(u+3\right)^{\frac{7}{3}}\left(3/2-u\right)^{\frac{11}{3}}}, & u<\dfrac{3}{2},\\ 0, & u>\dfrac{3}{2},\end{cases} \qquad (28)$$

where $u=R_0/\overline{R_0}$, R_0 and $\overline{R_0}$ – the QD radius and its mean value respectively; e – base of natural logarithm.

Coefficient of the light impurity absorption $\alpha^{(s)}(\omega)$ with longitudinal polarization to the magnetic field direction, taking into account the QDs size dispersion, is determined by the expression

$$\alpha^{(s)}(\omega)=\frac{2\pi N_0}{\hbar I_0}\sum_n\int_0^{3/2}du\,P(u)\left|M^{(s)}\right|^2\delta\left(E_{n,0,1}+\left|E_1\right|+\left|E_2\right|-\hbar\omega\right), \qquad (29)$$

where N_0 – the QDs concentration in dielectric matrix; $\delta(x)$ – the Dirac delta function.

Taking into account (27) and performing integration in (29), the light impurity absorption coefficient $\alpha^{(s)}(\omega)$ can be written as

$$\alpha^{(s)}(X)=\sum_{n=1}^{N}\sum_{l=0}^{K}\frac{P(u_n)}{X}\frac{\lambda_0^2\alpha^* N_0\pi^2 E_d}{a_d^2\hbar\left(\overline{R}_0^*u_n\right)^2\mathrm{sh}^6\left(\overline{R}_0^*u_n\eta^{-1}\right)}\times$$

$$\times\left(2\xi_{nl}^2\left(\overline{R}_0^*u_n\right)^{-2}+V_{n,l,0;n,l,0}+\sum_{n'l'm'}\frac{\left(\overline{R}_0^*u_n\right)^2\left|V_{n,l,0;n',l',m'}\right|^2}{\pi^2-\tilde{X}_{n',l'}^2}+\left|E_1\right|/E_d+\left|E_2\right|/E_d\right)^2\times$$

$$\times\frac{2^6\eta^{-3}}{\left(\mathrm{th}\left(\overline{R}_0^*u_n\eta^{-1}\right)-\overline{R}_0^*u_n\eta^{-1}\mathrm{cosec}\left(\overline{R}_0^*u_n\eta^{-1}\right)\right)^3}\times$$

$$\times\Bigg|\frac{\sqrt{k_{n1}^{-3}\left(k_{n1}^2+\eta^{-2}\right)^{-2}}\sqrt{k_{n1}+i\eta^{-1}}\left(k_{n1}-i2\eta^{-1}\right)}{J_{l+3/2}\left(\xi_{nl}\right)}\times$$

$$\times\mathrm{ch}\left(\bar{R}_0^* u_n \eta^{-1}\right) S\left(\sqrt{\frac{2\bar{R}_0^* u_n}{\pi}\left(k_{n1} - i\eta^{-1}\right)}\right) + i\sqrt{k_{n1} - i\eta^{-1}}\left(k_{n1} + i2\eta^{-1}\right)\mathrm{ch}\left(\bar{R}_0^* u_n \eta^{-1}\right)\times$$

$$\times\mathrm{Si}\left(\sqrt{\frac{2\bar{R}_0^* u_n}{\pi}\left(k_{n1} + i\eta^{-1}\right)}\right) + \sqrt{k_{n1} + i\eta^{-1}}\left(k_{n1} - i2\eta^{-1}\right)\mathrm{sh}\left(\bar{R}_0^* u_n \eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2\bar{R}_0^* u_n}{\pi}\left(k_{n1} - i\eta^{-1}\right)}\right) + \sqrt{k_{n1} - i\eta^{-1}}\left(k_{n1} + i2\eta^{-1}\right) sh\left(\bar{R}_0^* u_n \eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2}{\pi}\bar{R}_0^* u_n\left(k_{n1} + i\eta^{-1}\right)}\right)\Bigg]\sqrt{\frac{2}{\pi k_{n1}}}\left(\eta^{-2} + k_{n1}^2\right)^{-2}\times$$

$$\times\Big\{\eta^{-1}\Big[\left(\eta^{-2} + k_{n1}^2\right)\bar{R}_0^* u_n \cos\left(k_{n1}\bar{R}_0^* u_n\right) - \left(\eta^{-2} + 2k_{n1} + k_{n1}^2\right)\bar{R}_0^* u_n \sin\left(k_{n1}\bar{R}_0^* u_n\right)\Big] +$$

$$+\left(\eta^{-2}\left(k_{nl} - 1\right) + k_{nl}^2\left(k_{nl} + 1\right)\right) sh\left(\bar{R}_0^* u_n \eta^{-1}\right)\Big\} +$$

$$+\sum_{n'l'm'} \frac{\left(\bar{R}_0^* u_n\right)^2 V_{n,l,m;n',l',m'}}{\pi^2 - \tilde{X}_{n',l'}^2} \frac{\sqrt{k_{n'1}^{-3}\left(k_{n'1}^2 + \eta^{-2}\right)^{-2}}\sqrt{k_{n'1} + i\eta^{-1}}\left(k_{n'1} - i2\eta^{-1}\right)}{J_{l'+3/2}\left(\xi_{n'l}\right)}\times$$

$$\times\mathrm{ch}\left(\bar{R}_0^* u_n \eta^{-1}\right) S\left(\sqrt{\frac{2\bar{R}_0^* u_n}{\pi}\left(k_{n'1} - i\eta^{-1}\right)}\right) + i\sqrt{k_{n'1} - i\eta^{-1}}\left(k_{n'1} + i2\eta^{-1}\right)\mathrm{ch}\left(\bar{R}_0^* u_n \eta^{-1}\right)\times$$

$$\times\mathrm{Si}\left(\sqrt{\frac{2\bar{R}_0^* u_n}{\pi}\left(k_{n'1} + i\eta^{-1}\right)}\right) + \sqrt{k_{n'1} + i\eta^{-1}}\left(k_{n'1} - i2\eta^{-1}\right)\mathrm{sh}\left(\bar{R}_0^* u_n \eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2\bar{R}_0^* u_n}{\pi}\left(k_{n'1} - i\eta^{-1}\right)}\right) + \sqrt{k_{n'1} - i\eta^{-1}}\left(k_{n'1} + i2\eta^{-1}\right)\mathrm{sh}\left(\bar{R}_0^* u_n \eta^{-1}\right)\times$$

$$\times Ci\left(\sqrt{\frac{2\bar{R}_0^* u_n}{\pi}\left(k_{n'1} + i\eta^{-1}\right)}\right)\Bigg]\sqrt{\frac{2}{\pi k_{n'1}}}\left(\eta^{-2} + k_{n'1}^2\right)^{-2}\times$$

$$\times\Big\{\eta^{-1}\Big[\left(\eta^{-2} + k_{n'1}^2\right)\bar{R}_0^* u_n \cos\left(k_{n'1}\bar{R}_0^* u_n\right) - \left(\eta^{-2} + 2k_{n'1} + k_{n''1}^2\right)\bar{R}_0^* u_n \sin\left(k_{n'1}\bar{R}_0^* u_n\right)\Big] +$$

$$+\left(\eta^{-2}\left(k_{n'l'} - 1\right) + k_{n'l'}^2\left(k_{n'l'} + 1\right)\right)\mathrm{sh}\left(\bar{R}_0^* u_n \eta^{-1}\right)\Big\}\Big|^2, \qquad (30)$$

where

$$u_n = \left[2\left(X - |E_1|/E_d - |E_2|/E_d - V_{n,1,-1;n,1,-1}/E_d \right) \right]^{-\frac{1}{2}} \times$$

$$\times \left(\bar{R}_0^{*2} \sum_{n'l'm'} \left| V_{n,1,-1;n',l',m'} \right|^2 \Big/ E_d \left(\tilde{X}_{n',l'}^2 - \pi^2 \right) + \right.$$

$$+ \left[\left(\bar{R}_0^{*2} \sum_{n'l'm'} \left| V_{n,1,-1;n',l',m'} \right|^2 \Big/ E_d \left(\tilde{X}_{n',l'}^2 - \pi^2 \right) \right)^2 + \right.$$

$$\left. \left. +4\left(X - |E_1| - |E_2| - V_{n,1,-1;n,1,-1} \right) 2\xi_{n1}^2 \Big/ E_d \bar{R}_0^{*2} \right]^{\frac{1}{2}} \right)^{\frac{1}{2}} ; X = \hbar\omega / E_d .$$

Next we consider the double photoionization process for TIC in an external magnetic field in a semiconductive QD in case of the light transverse polarization to the external magnetic field direction.

The effective Hamiltonian $\mathbf{H}_{\text{int}B}^{(t)}$ for interaction with the light wave field, which are characterized by a wave vector $\mathbf{q}$ and by the unit vector of polarization $\mathbf{e}_{\lambda t}$ transverse to the magnetic field direction can be written as follows

$$\mathbf{H}_{\text{int}B}^{(t)} = -i\hbar\lambda_0 \sqrt{\frac{2\pi\hbar^2\alpha^*}{m^{*2}\omega} I_0} \exp\left(i\mathbf{q}_t \mathbf{r}\right) \left(\left(\mathbf{e}_{\lambda t} \nabla_{\mathbf{r}} \right) - \frac{i|e|B}{2\hbar} \left[\mathbf{e}_{\lambda t}, \mathbf{r} \right]_z \right). \quad (31)$$

The matrix elements $M_{f\,QD,\lambda_B}^{(t)}$, determining the electron optical transitions from the ground state of TIC to the QD discrete spectrum states for case of the transverse polarization $\mathbf{e}_{\lambda t}$ can be represented in the dipole approximation as the sum of two terms:

$$M^{(t)} = M_1^{(t)} + M_2^{(t)}, \quad (32)$$

where

$$M_1^{(t)} = i\lambda_0 \sqrt{\frac{2\pi\alpha^*}{\omega} I_0} \left[\left(E_{n,l,m} - E_1 \right) \times \right.$$

$$\times \left\langle \Psi_{n,l,m}^*(\rho_1,\theta_1,\varphi_1)\psi^*(\rho_2) \right| \left(\mathbf{e}_{\lambda t}, \mathbf{r}_1 \right) \left| \Psi(\rho_1,\rho_2) \right\rangle +$$

$$\left. + \left(E_{n,l,m} - E_2 \right) \left\langle \Psi_{n,l,m}^*(\rho_1,\theta_1,\varphi_1)\psi^*(\rho_2) \right| \left(\mathbf{e}_{\lambda t}, \mathbf{r}_2 \right) \left| \Psi(\rho_1,\rho_2) \right\rangle \right] \quad (33)$$

and

$$M_2^{(t)} = -\lambda_0 \sqrt{\frac{2\pi\alpha^*}{\omega} I_0} \frac{\hbar\omega_B}{2} \left[\left\langle \Psi_{n,l,m}^*(\rho_1,\theta_1,\varphi_1)\psi^*(\rho_2) \right| \left[\mathbf{e}_{\lambda t}, \mathbf{r}_1 \right]_z \left| \Psi(\rho_1,\rho_2) \right\rangle + \right.$$

$$+\left\langle \Psi^{*}_{n,l,m}(\rho_1,\theta_1,\varphi_1)\psi^{*}(\rho_2)\right|\left[\mathbf{e}_{\lambda t},\mathbf{r}_2\right]_z\left|\Psi(\rho_1,\rho_2)\right\rangle\Big]. \qquad (34)$$

Calculations in (33) and (34) lead to the following expression for the square modulus of the matrix element (32)

$$\left|M^{(t)}\right|^2=\frac{\alpha^*}{\omega}\frac{\pi\lambda_0^2 I_0 E_d^2}{a_d^2 R_0^{*2} sh^6\left(R_0^*\eta^{-1}\right)}\times\frac{2^5\eta^{-3}}{\left(\mathrm{th}\left(R_0^*\eta^{-1}\right)-R_0^*\eta^{-1}\mathrm{cosec}\left(R_0^*\eta^{-1}\right)\right)^3}\times$$

$$\times\left(2\xi_{nl}^2 R_0^{*-2}+V_{n,l,m;n,l,m}+\sum_{n'l'm'}\frac{R_0^{*2}\left|V_{n,l,m;n',l',m'}\right|^2}{\pi^2-\tilde{X}^2_{n',l'}}+\right.$$

$$\left.+\left|E_1\right|/E_d+\left|E_2\right|/E_d+\hbar\omega_B/E_d\right)^2\times$$

$$\times\left|\frac{\sqrt{k_{n1}^{-3}\left(k_{n1}^2+\eta^{-2}\right)^{-2}}\sqrt{k_{n1}+i\eta^{-1}}\left(k_{n1}-i2\eta^{-1}\right)}{J_{l+3/2}(\xi_{nl})}\times\right.$$

$$\times\mathrm{ch}\left(R_0^*\eta^{-1}\right)S\left(\sqrt{\frac{2}{\pi}}R_0^*\left(k_{n1}-i\eta^{-1}\right)\right)+i\sqrt{k_{n1}-i\eta^{-1}}\left(k_{n1}+i2\eta^{-1}\right)\mathrm{ch}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Si}\left(\sqrt{\frac{2}{\pi}}R_0^*\left(k_{n1}+i\eta^{-1}\right)\right)+\sqrt{k_{n1}+i\eta^{-1}}\left(k_{n1}-i2\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2}{\pi}}R_0^*\left(k_{n1}-i\eta^{-1}\right)\right)+\sqrt{k_{n1}-i\eta^{-1}}\left(k_{n1}+i2\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\times$$

$$\times\mathrm{Ci}\left(\sqrt{\frac{2}{\pi}}R_0^*\left(k_{n1}+i\eta^{-1}\right)\right)\Bigg]\sqrt{\frac{2}{\pi k_{n1}}}\left(\eta^{-2}+k_{n1}^2\right)^{-2}\times$$

$$\times\left\{\eta^{-1}\left[\left(\eta^{-2}+k_{n1}^2\right)R_0^*\cos\left(k_{n1}R_0^*\right)-\left(\eta^{-2}+2k_{n1}+k_{n1}^2\right)R_0^*\sin\left(k_{n1}R_0^*\right)\right]+\right.$$

$$\left.+\left(\eta^{-2}\left(k_{nl}-1\right)+k_{nl}^2\left(k_{nl}+1\right)\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\right\}+$$

$$+\sum_{n'l'm'}\frac{R_0^{*2}V_{n,l,m;n',l',m'}}{\pi^2-\tilde{X}^2_{n',l'}}\frac{\sqrt{k_{n'1}^{-3}\left(k_{n'1}^2+\eta^{-2}\right)^{-2}}\sqrt{k_{n'1}+i\eta^{-1}}\left(k_{n'1}-i2\eta^{-1}\right)}{J_{l'+3/2}(\xi_{n'l})}\times$$

$$\times \mathrm{ch}\left(R_0^*\eta^{-1}\right)S\left(\sqrt{\frac{2R_0^*}{\pi}\left(k_{n'1}-i\eta^{-1}\right)}\right)+i\sqrt{k_{n'1}-i\eta^{-1}}\left(k_{n'1}+i2\eta^{-1}\right)\mathrm{ch}\left(R_0^*\eta^{-1}\right)\times$$

$$\times \mathrm{Si}\left(\sqrt{\frac{2R_0^*}{\pi}\left(k_{n'1}+i\eta^{-1}\right)}\right)+\sqrt{k_{n'1}+i\eta^{-1}}\left(k_{n'1}-i2\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\times$$

$$\times \mathrm{Ci}\left(\sqrt{\frac{2R_0^*}{\pi}\left(k_{n'1}-i\eta^{-1}\right)}\right)+\sqrt{k_{n'1}-i\eta^{-1}}\left(k_{n'1}+i2\eta^{-1}\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\times$$

$$\times \mathrm{Ci}\left(\sqrt{\frac{2R_0^*}{\pi}\left(k_{n'1}+i\eta^{-1}\right)}\right)\Bigg]\sqrt{\frac{2}{\pi k_{n'1}}}\left(\eta^{-2}+k_{n'1}^2\right)^{-2}\times$$

$$\times\Big\{\eta^{-1}\Big[\left(\eta^{-2}+k_{n'1}^2\right)R_0^*\cos\left(k_{n'1}R_0^*\right)-\left(\eta^{-2}+2k_{n'1}+k_{n''1}^2\right)R_0^*\sin\left(k_{n'1}R_0^*\right)\Big]+$$

$$+\left(\eta^{-2}\left(k_{n'l'}-1\right)+k_{n'l'}^2\left(k_{n'l'}+1\right)\right)\mathrm{sh}\left(R_0^*\eta^{-1}\right)\Big\}\Big|^2. \qquad (35)$$

In this case, calculation in (33) and (36) of the following integral

$$\int_0^{2\pi} d\varphi\cos\varphi\exp(-im\varphi)=\begin{cases}\pi,\ if\ m=\pm1,\\ 0,\ if\ m\neq\pm1\end{cases} \qquad (36)$$

gives a selection rule for the magnetic quantum number m.

Assuming that the QDs-size distribution function is still the Lifshitz-Slezov function (28), expression for the light absorption coefficient $\alpha_B^{(t)}(\omega)$ can be written as

$$\alpha_B^{(t)}(\omega)=\frac{2\pi N_0}{\hbar I_0}\sum_{n_1,n}\sum_{m=-1}^{1}\delta_{|m|,1}\int_0^{\frac{3}{2}}du\ P(u)\left|M^{(t)}\right|^2\delta\left(E_{n,l,1}+|E_1|+|E_2|-\hbar\omega\right), \qquad (37)$$

where $\delta_{|m|,1}$ – the Kronecker symbol, taking into account the selection rule for a magnetic quantum number m (36):

$$\alpha^{(t)}(X)=\frac{\lambda_0^2\alpha^* N_0 2^6\pi^2 E_d}{a_d^2\hbar}\sum_{n=1}^{N}\sum_{l=0}^{K}\sum_{m=-1}^{+1}\delta_{m,\pm1}\frac{P(u_n)}{XR_0^{*2}sh^6\left(\bar{R}_0^*u_n'\eta^{-1}\right)}\times$$

$$\times\left(2\xi_{nl}^2\left(\bar{R}_0^*u_n'\right)^{-2}+V_{n,l,m;n,l,m}+\sum_{n'l'm'}\frac{\left(\bar{R}_0^*u_n'\right)^2\left|V_{n,l,m;n',l',m'}\right|^2}{\pi^2-\tilde{X}_{n',l'}^2}+\right.$$

$$\left. +|E_1|/E_d + |E_2|/E_d + \hbar\omega_B/E_d \right)^2 \times$$

$$\times \frac{\eta^{-3}}{\left(\operatorname{th}\left(\bar{R}_0^* u_n' \eta^{-1}\right) - \bar{R}_0^* u_n' \eta^{-1} \operatorname{cosec}\left(\bar{R}_0^* u_n' \eta^{-1}\right)\right)^3} \times$$

$$\times \left| \frac{\sqrt{k_{n1}^{-3}\left(k_{n1}^2 + \eta^{-2}\right)^{-2}} \sqrt{k_{n1} + i\eta^{-1}} \left(k_{n1} - i2\eta^{-1}\right)}{J_{l+3/2}\left(\xi_{nl}\right)} \times \right.$$

$$\times \operatorname{ch}\left(\bar{R}_0^* u_n' \eta^{-1}\right) S\left(\sqrt{\frac{2\bar{R}_0^* u_n'}{\pi}\left(k_{n1} - i\eta^{-1}\right)}\right) + i\sqrt{k_{n1} - i\eta^{-1}}\left(k_{n1} + i2\eta^{-1}\right) \operatorname{ch}\left(\bar{R}_0^* u_n' \eta^{-1}\right) \times$$

$$\times \operatorname{Si}\left(\sqrt{\frac{2\bar{R}_0^* u_n'}{\pi}\left(k_{n1} + i\eta^{-1}\right)}\right) + \sqrt{k_{n1} + i\eta^{-1}}\left(k_{n1} - i2\eta^{-1}\right) \operatorname{sh}\left(\frac{\bar{R}_0^* u_n'}{\eta}\right) \times$$

$$\times \operatorname{Ci}\left(\sqrt{\frac{2\bar{R}_0^* u_n'}{\pi}\left(k_{n1} - i\eta^{-1}\right)}\right) + \sqrt{k_{n1} - i\eta^{-1}}\left(k_{n1} + i2\eta^{-1}\right) \operatorname{sh}\left(\bar{R}_0^* u_n' \eta^{-1}\right) \times$$

$$\left. \times \operatorname{Ci}\left(\sqrt{\frac{2}{\pi} \bar{R}_0^* u_n' \left(k_{n1} + i\eta^{-1}\right)}\right)\right] \sqrt{\frac{2}{\pi k_{n1}}} \left(\eta^{-2} + k_{n1}^2\right)^{-2} \times$$

$$\times \left\{ \eta^{-1}\left[\left(\eta^{-2} + k_{n1}^2\right) \bar{R}_0^* u_n' \cos\left(k_{n1} \bar{R}_0^* u_n'\right) - \left(\eta^{-2} + 2k_{n1} + k_{n1}^2\right) \bar{R}_0^* u' \sin\left(k_{n1} \bar{R}_0^* u_n'\right)\right] + \right.$$

$$\left. + \left(\eta^{-2}\left(k_{nl} - 1\right) + k_{nl}^2\left(k_{nl} + 1\right)\right) \operatorname{sh}\left(R_0^* u_n' \eta^{-1}\right)\right\} +$$

$$+ \sum_{n'l'm'} \frac{\left(\bar{R}_0^* u_n'\right)^2 V_{n,l,m;n',l',m'}}{\pi^2 - \tilde{X}_{n',l'}^2} \frac{\sqrt{k_{n'1}^{-3}\left(k_{n'1}^2 + \eta^{-2}\right)^{-2}} \sqrt{k_{n'1} + i\eta^{-1}} \left(k_{n'1} - i2\eta^{-1}\right)}{J_{l'+3/2}\left(\xi_{n'l}\right)} \times$$

$$\times \operatorname{ch}\left(\bar{R}_0^* u_n' \eta^{-1}\right) S\left(\sqrt{\frac{2\bar{R}_0^* u_n'}{\pi}\left(k_{n'1} - i\eta^{-1}\right)}\right) + i\sqrt{k_{n'1} - i\eta^{-1}}\left(k_{n'1} + i2\eta^{-1}\right) \times$$

$$\times \operatorname{ch}\left(\bar{R}_0^* u_n' \eta^{-1}\right) \operatorname{Si}\left(\sqrt{\frac{2\bar{R}_0^* u_n'}{\pi}\left(k_{n'1} + i\eta^{-1}\right)}\right) + \sqrt{k_{n'1} + i\eta^{-1}}\left(k_{n'1} - i2\eta^{-1}\right) \operatorname{sh}\left(\frac{\bar{R}_0^* u_n'}{\eta}\right) \times$$

$$\times \operatorname{Ci}\left(\sqrt{\frac{2\bar{R}_0^* u_n'}{\pi}\left(k_{n'1} - i\eta^{-1}\right)}\right) + \sqrt{k_{n'1} - i\eta^{-1}}\left(k_{n'1} + i2\eta^{-1}\right) \operatorname{sh}\left(\bar{R}_0^* u_n' \eta^{-1}\right) \times$$

$$\times \mathrm{Ci}\left(\sqrt{\frac{2\bar{R}_0^* u_n'}{\pi}\left(k_{n'1}+i\eta^{-1}\right)}\right)\Bigg]\sqrt{\frac{2}{\pi k_{n'1}}}\left(\eta^{-2}+k_{n'1}^2\right)^{-2}\times$$

$$\times\Big\{\eta^{-1}\Big[\left(\eta^{-2}+k_{n'1}^2\right)\bar{R}_0^* u_n' \cos\left(k_{n'1}\bar{R}_0^* u_n'\right)-\left(\eta^{-2}+2k_{n'1}+k_{n'1}^2\right)\times$$

$$\times \bar{R}_0^* u_n' \sin\left(k_{n'1}\bar{R}_0^* u_n'\right)+\left(\eta^{-2}\left(k_{n'l'}-1\right)+k_{n'l'}^2\left(k_{n'l'}+1\right)\right)\mathrm{sh}\left(\bar{R}_0^* u_n' \eta^{-1}\right)\Big\}\Bigg|^2, \quad (38)$$

where

$$u_n' = \left[2\left(X-|E_1|/E_d-|E_2|/E_d-V_{n,1,-1;n,1,-1}/E_d\right)\right]^{-\frac{1}{2}}\times$$

$$\times\left(\bar{R}_0^{*2}\sum_{n'l'm'}\left|V_{n,1,-1;n',l',m'}\right|^2\Big/E_d\left(\tilde{X}_{n',l'}^2-\pi^2\right)+\right.$$

$$+\left[\left(\bar{R}_0^{*2}\sum_{n'l'm'}\left|V_{n,1,-1;n',l',m'}\right|^2\Big/E_d\left(\tilde{X}_{n',l'}^2-\pi^2\right)\right)^2+\right.$$

$$\left.\left.+4\left(X-|E_1|-|E_2|-V_{n,1,-1;n,1,-1}-\hbar\omega_B\right)2\xi_{n1}^2\Big/E_d\bar{R}_0^{*2}\right]^{\frac{1}{2}}\right)^{1/2}.$$

Figure 2 shows the spectral dependences of the light absorption coefficients associated with double photoionization of TIC in an external magnetic field.

It can be seen that in case of $\vec{e}_\lambda \perp \vec{B}$ (curve 2) the light impurity absorption threshold shifts to the short-wave region of the spectrum due to increase in energy of the TIC ground state. In addition to the main "double-humped" profile on the absorption curve, additional peaks appear (Zeeman doublet).

The same figure shows dynamics of the impurity absorption band with an increase in the magnetic field induction B in case of $\vec{e}_\lambda \parallel \vec{B}$ (cf. curves 1 and 3): the absorption threshold shifts to the long-wavelength region of the spectrum, while the absorption coefficient decreases and the right peak on the spectral curve disappears, which associated with effect of the electronic correlations suppressing in a magnetic field.

Thus, the optical absorption dichroism is observed in the double photoionization spectra of TIC in a magnetic field, which is associated with change in the selection rules for a magnetic quantum number m: $m=0$ in case $\vec{e}_\lambda \parallel \vec{B}$ and $m=\pm 1$ in case $\vec{e}_\lambda \perp \vec{B}$.

Conclusions

Theoretical study of the magnetic field influence effect on the binding energy for TIC has been carried out by variational method in framework of the perturbation theory. It is shown that with an increase in an external magnetic field, an increase in the TIC binding energy occurs and magnetic field has a stabilizing effect on TIC in semiconductive QD due to effect of the electronic correlations suppressing. In the dipole approximation, in framework of the effective mass method, the light impurity absorption coefficients in case of double ionization for TIC by a single photon in an external magnetic field, have been calculated.

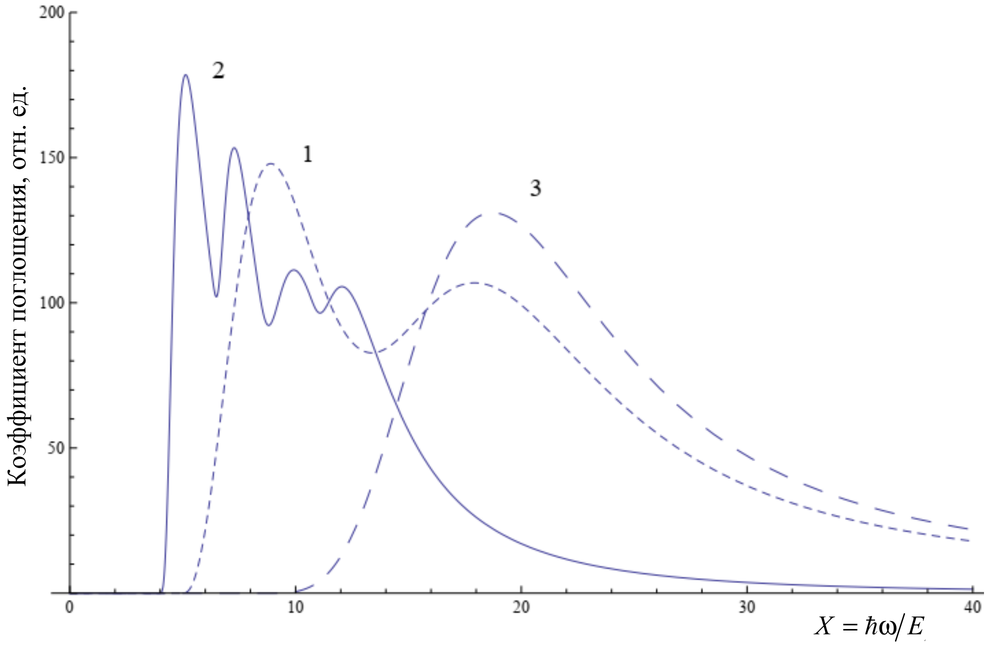

Fig. 2. Spectral dependence of the light absorption coefficients in the double photoionization of two electron impurity centers in a quasi-zero-dimensional structure at $E = 0,03$ eV; $\bar{R} = 1$, for different directions of the light polarization relative to an external magnetic field: B: 1 – $B = 0$, 2 – $B = 2\,T$ ($\vec{e} \perp \vec{B}$), 3 – $B = 2$ T ($\vec{e} \parallel \vec{B}$)

It is shown that the light optical absorption dichroism is associated with change in the selection rules for a magnetic quantum number. In case of $\vec{e}_\lambda \parallel \vec{B}$, the magnetic field influence is appeared in an increase of the photon energy threshold value and in disappearance of the right peak on the spectral curve due to suppression of electron correlations in a magnetic field.

A characteristic feature of the double photoionization spectrum in case of $\vec{e}_\lambda \perp \vec{B}$ is the Zeeman double doublet: four additional peaks appear on the main "humps" of the absorption curve, while the light impurity absorption band edge is shifted to the spectrum long-wave region.

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Кревчик Владимир Дмитриевич
доктор физико-математических наук, профессор, декан факультета приборостроения, информационных технологий и электроники, Пензенский государственный университет (Россия, г. Пенза, ул. Красная, 40)

E-mail: physics@pnzgu.ru

Krevchik Vladimir Dmitrievich
Doctor of physical and mathematical sciences, professor, dean of the faculty of instrument engineering, information technology and electronics, Penza State University (40 Krasnaya street, Penza, Russia)

Разумов Алексей Викторович
кандидат физико-математических наук, доцент, кафедра общей физики и методики обучения физике, Пензенский государственный университет (Россия, г. Пенза, ул. Красная, 40)

E-mail: Razumov_alex@mail.ru.

Razumov Aleksey Viktorovich
Candidate of physical and mathematical sciences, associate professor, sub-department of general physics and methods of physics teaching, Penza State University (40 Krasnaya street, Penza, Russia)

Мойко Ирина Михайловна
аспирант, Пензенский государственный университет (Россия, г. Пенза, ул. Красная, 40)

E-mail: irmojko@yandex.ru

Moyko Irina Mikhaylovna
Postgraduate student, Penza State University (40 Krasnaya street, Penza, Russia)

Ли Тян-Ронг
доктор философии, доцент, Институт функциональных и экологических материалов, Университет Ланчжоу (КНР, Ланчжоу, ул. Саут Тяншуй 222)

E-mail: litr@lzu.edu.cn

Li Tian-Rong
PhD, associate professor, Institute of Functional and Environmental Materials, Lanzhou University (222 South Tianshui South Road, Lanzhou, P. R. China)

Ванг Ю-Хуа
доктор философии, Университет Ланчжоу, директор Международного Управления, директор Управления по связям с Гонконгом, Макао и Тайванем, Профессор кафедры исследования люминесцентных материалов, Школа Физических Наук и Технологии, директор Центральной Лаборатории Специальных Функциональных Материалов и Структурного Дизайна, Министерство Образования (КНР, Ланчжоу, ул. Саут Тяншуй 222)

E-mail: wyh@lzu.edu.cn

Wang Yu-Hua
PhD, Lanzhou University Director of International Office Director of the Office of Hong Kong, Macao and Taiwan Affairs Professor of Luminescent Materials Studies in School of Physical Science and Technology Director of the Key Laboratory of Special Function Materials and Structure Design, the Ministry of Education (222 South Tianshui South Road, Lanzhou, P. R. China)

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