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# About Aleksandr Semenovich Kronrod

Aleksandr Semenovich Kronrod was born on 22 October 1921 in Moscow.

Sasha Kronrod was introduced to mathematics in the now almost legendary school study group of the Moscow State University (MSU) student D. O. Shklyarskii, a talented beginning scientist and brilliant teacher who had his pupils focus on the independent solution of difficult problems.[1] In 1938 Kronrod enrolled in the Mechanics and Mathematics Department (MechMat) of MSU, and soon became known to all the department's students and lecturers because of his obviously outstanding abilities, his extraordinary activity, energy, and temperament, his often deliberately pointed and paradoxical remarks, and even his purely external features: his unusual height, his beautiful sonorous voice.

Kronrod carried out his first independent work already in the first year: A. O. Gel'fond, who headed the mathematical analysis section in those years and supervised the circle for beginning students, posed to him the problem of describing the possible structure of the set of points of discontinuity of a function that is differentiable where it is continuous, a fairly traditional problem for prewar Moscow mathematics (but not for Gel'fond himself), and in 1939 Kronrod's solution of this problem appeared in *Izvestiya Akademii Nauk* as his first research paper.

The normal course of life for students of Kronrod's generation was interrupted by the war. On the first day of the war he put in a request to the military recruitment office asking to be sent to the front, but he was refused: students of the higher classes were given deferments. Later they were sent to study in military academies, and in the first days of the war to help build defences around Moscow.[2] Upon returning from this duty Kronrod again began to assault the recruitment office and finally got his way: he was taken into the real army.

The military path of the Soviet army officer Kronrod was not easy: at the time of the winter offensive of our army near Moscow his bravery was distinguished not only by a first decoration but also by a first serious wound. After a second wound in 1943 a return to his unit was out of the question: Kronrod still had his ability

[1]Kronrod received a first prize in the 1938 fourth Moscow mathematical olympiad. (This and the subsequent footnotes were added by the Russian editor.)

[2] ... where Kronrod led the student brigade from MechMat.

to do mathematics, but not to wage war. The second wound had left him disabled, and he felt the effects for the rest of his years.

In a hospital Kronrod turned to a problem that M. A. Kreines had posed to him before the war. The problem was the following: suppose that a permutation $i \to k_i$ of the set $\mathbb{N} = \{i\} = \{1, 2, 3, \dots\}$ of natural numbers changes the sum of some infinite series, $\sum_{i=1}^{\infty} a_i \neq \sum a_{k_i}$; determine whether there is then a (conditionally) convergent series $\sum b_i$ that is carried by the same permutation into a divergent series.

Kronrod substantially extended the framework of the problem posed to him: he was able to show that, with respect to their action on (conditionally convergent) series, all permutations can be divided into several categories. There are permutations carrying some convergent series into divergent series, and he called these 'left' permutations. He called permutations carrying some divergent series into a convergent series 'right' permutations; it was clear that the permutation inverse to a left permutation is always a right permutation. The intersection of the sets of left and right permutations form the 'two-sided' permutations: they can carry a convergent series into a divergent series and also a divergent series into a convergent series. Permutations that are neither left nor right he called 'neutral'—these permutations cannot change the convergence of a series, and it turned out that they cannot change the sum of any series. The last point follows from the fact that, as became clear, the set of permutations that can change sums of series (Kronrod called them 'essential') is a subset of the set of two-sided permutations.

The concluding part of the paper contained effective criteria for a permutation to belong to the various classes (left, right, two-sided, neutral, essential) and an extension of the main results to series with complex terms.

This very beautiful paper, published in 1945 in *Matematicheskii Sbornik*, served as Kronrod's diploma work, and for it he was also awarded the Moscow Mathematical Society (MMS) prize for young researchers (it was almost unheard of, and in any case very rare, for a student to be awarded the MMS prize; moreover, Kronrod was the only person to receive this highly prestigious prize twice).

In the fall of 1944 Kronrod renewed his studies as a fourth-year student in the Mechanics and Mathematics Department. And in February of 1945 a noteworthy event took place in the department: after a long interruption Academician N. N. Luzin again began giving lectures, announcing a course on the theory of functions of two real variables. He also started a seminar closely related to this course.

In those years Nikolai Nikolaevich Luzin was almost a mythical figure for students of the university. Nearly all the leading mathematics professors of the older and middle generations had been students of Luzin. There were legends about the famous 'Luzitania' (an association of students of Luzin). However, since Luzin had not taught in recent years, a natural break had formed in the ranks of his students. Kronrod and G. M. Adel'son-Vel'skii should evidently be regarded as his last students, appearing after the long break. Furthermore, while Adel'son-Vel'skii had other teachers in addition to Luzin (I. M. Gel'fand, and in the area of computational mathematics Kronrod, who was only a little older than him), Kronrod's only teacher was Luzin. Kronrod always took pride in this his student period—he liked to show off a copy of the French original of Luzin's famous dissertation "The Integral and Trigonometric Series" given to him and signed by its author, and he recalled with pleasure how Luzin had introduced him to the celebrated J. Hadamard as his student.

It seems that the strongest side of Luzin was always his ability to give his students problems of great significance in general mathematics, problems whose methods of solution, worked out independently by strong and persistent young researchers, led to the emergence of new directions.

The problem he posed to Adel'son-Vel'skii and Kronrod was to prove the analyticity of a monogenic function by methods in the theory of functions of a real variable without using the Cauchy integral nor the theory of functions of a complex variable: it was required to establish that each function $w(x,y) = u(x,y) + iv(x,y)$ with $u(x,y)$ and $v(x,y)$ satisfying the Cauchy–Riemann conditions can be expanded in a convergent power series.

Adel'son-Vel'skii and Kronrod solved the problem, and even generalized it: they looked at arbitrary equations

$$\frac{\partial u}{\partial x} = A(x,y)\frac{\partial v}{\partial y}\,, \qquad \frac{\partial u}{\partial y} = -B(x,y)\frac{\partial v}{\partial x}$$

with positive functions and established a connection between smoothness of the solutions and smoothness of the coefficients (identically equal to 1 in the case of the Cauchy–Riemann equations). An essential role in their work was played by a study of level curves of functions of two variables and a proof of the maximum principle for these functions.

This work served as a starting point for a study of level curves of arbitrary (continuous) functions of two variables, and Kronrod and Adel'son-Vel'skii devoted a series of subsequent papers to the topic.

But Kronrod did not confine himself to this—it was not in his nature to pose special problems: we shall have occasion below to discuss his maximalism (in life as well as in science). In his encounters with functions of two variables Kronrod discovered that while the theory of (continuous) functions of a single (real) variable had by that time been brought to a state of some degree of completeness, a theory simply did not exist for functions of two (and more) variables. Here the simplest concepts from the theory of functions of a single variable had been carried over, in a form not containing anything 'essentially two-dimensional'. Since a theory of functions of two variables did not exist, it was necessary to create one. In the course of the next few years Kronrod turned his whole attention to the solution of this important problem.

Over the next four years he constructed a harmonious theory encompassing the properties of functions of two real variables and involving the concept of variation, and he had in view a way to construct a theory of functions of several variables.

From the very beginning he rejected definitions connected with a specific orthogonal coordinate system (for example, the variation in the sense of Tonelli) and introduced concepts invariant with respect to orthogonal transformations. Variations of functions of two variables were fundamental concepts for his theory. He showed that properties depending on the variation in the case of a function of one variable split upon passing to a function of two variables, so that for a function of two variables it was natural to introduce two variations. One of them he called the *planar* variation, and the other the *linear* variation. Boundedness of the planar variation ensures that the function has an asymptotic total differential almost everywhere. For a smooth function it turns out to be equal to the integral of the absolute value of its gradient, taken over its domain.

The linear variation was a fundamentally new object. Kronrod introduced a concept of monotonicity for a function of two variables that generalized in a natural way the concept of monotonicity for a function of one variable, and he proved that boundedness of the linear variation enabled him to represent the function as a difference of two monotone functions. For the linear variation itself he gave a number of equivalent definitions, of which one is especially interesting. It turns out that with a continuous function of two variables one can associate a one-dimensional tree whose elements are the components of the level sets of the function. With the help of the original function a metric is given on this tree, on which a function is also defined. The linear variation then turns out to be equal to the usual variation of the function defined on the one-dimensional tree. Boundedness of both the planar and the linear variation ensures that the usual total differential exists almost everywhere.

Kronrod was considering continuous functions; however, the concepts he introduced carry over easily to discontinuous functions. He had in view also a programme of investigation of functions of several variables, and his students subsequently realized it.

At this time Kronrod became the centre of an active group of students (his pedagogical activities will be discussed at greater length later) who implemented many of his ideas (for example, A. G. Vitushkin constructed a theory of variations for functions of several variables and for set functions, and A. Ya. Dubovitskii investigated in detail the sets of critical values of functions of several variables and of smooth maps from $\mathbb{R}^n$ to $\mathbb{R}^m$; in particular, he re-proved Sard's theorem, at the time not yet known in Moscow, and he obtained several more subtle theorems about the structure of the set of critical points).

From today's point of view, after half a century, it is not Kronrod's results themselves that are of greatest interest (though an important stage, it is one that has passed), but the apparatus he developed to obtain them. For instance, his one-dimensional tree was used by V. I. Arnol'd in solving Hilbert's thirteenth problem.

However, the following theorem of Kronrod is especially well known in our day.

Let $G \subset \mathbb{R}^n$ be a domain, $f\colon G \to \mathbb{R}^1$ a smooth function, $E_t = \{x \in G \mid f(x) = t\}$ a level set of $f$, and $ds$ the $(n-1)$-dimensional area element of $E_t$. Then

$$\operatorname{meas} G = \int_{\min f}^{\max f} \left( \int_{E_t} \frac{ds}{|\operatorname{grad} f|} \right) dt.$$

For example, this theorem lies at the basis of many modern proofs in the theory of partial differential equations.

Kronrod's theory of functions of two variables made up the content of his dissertation, which he defended at MSU in 1949. The official opponents for the dissertation were M. V. Keldysh, A. N. Kolmogorov, and D. E. Men'shov. For this work Kronrod was awarded straightaway the degree of doctor of the physical and mathematical sciences, skipping the candidate's degree.[3]

The next important problem undertaken by Kronrod was the following. Let $S$ be a parametrically defined surface embedded in $\mathbb{R}^3$ and having bounded area in the Lebesgue sense. Determine whether $S$ has an asymptotic tangent plane almost everywhere (in the sense of the measure generated on $S$ by the Lebesgue area). At that time this was already a longstanding problem.

Kronrod solved it in the positive sense, but he did not publish the solution. This was because he had decided to make a break with pure mathematics. He did this decisively and forever.

To understand what happened one has to go back a bit. From 1945, his fourth year, Kronrod began working in the computation section of the Kurchatov Institute, in parallel with his studies. Initially it was material considerations that led him to do this: he had married, and a son was born in 1943. In particular, he needed a place to live. Working at the Kurchatov Institute gave such an opportunity. But Kronrod never did anything in a halfhearted manner. As soon as he came into contact with computational mathematics, he began studying it in earnest. And he discovered that this was an interesting area, quite unlike pure mathematics, he thought. He always stressed that methods of computation had to be distinguished from the theorems proved about computational mathematics. For example, he said that in solving a differential equation by the finite-difference method it was necessary to make up the finite-difference scheme starting out from the physical problem and not from the differential equation, and that one should never be concerned with whether or not the solution of the finite-difference equation converges to the solution of the differential equation, for if the construction of the scheme is physically correct and convergence to the solution of the differential equation fails, then so much the worse for the differential equation. As a rule, one should not make theoretical estimates of the error. Such an estimate requires describing the supply of functions among which the solution is found. But this supply (like the distribution of solutions in it) is unknown to us a priori. This is now all commonplace, but at that time it sounded paradoxical. Kronrod devised a number of algorithms for rapidly solving various problems; in particular, he discovered the sweep method independently of several other researchers.

Thus, Kronrod had found a new sphere of activity for himself. This alone would not likely have been sufficient for such a decisive break with traditional mathematics, despite the maximalism, already mentioned above, which was one of the leading traits of his character.

In addition to electric calculating machines—'Mercedes' machines—the computational technology of the time included tabulators and sorters, which operated with punched cards. Fate threw Kronrod together with the talented engineer and relay exponent Nikolai Ivanovich Bessonov. Using several tabulators and a supplementary relay device he had built for multiplication of numbers, Bessonov constructed

[3]It was published in his survey in *Uspekhi Matematicheskikh Nauk* [15].

a multi-purpose machine on which more complex computational problems could be solved. Here Kronrod and Bessonov (evidently, the logic side of the problem was Kronrod's, while the design side was undoubtedly Bessonov's) gave birth to the idea of a universal digital computing machine controlled by a program. It should be recalled that at the time (at the beginning of the second half of the 1940s) we did not yet know of the existence of the American electronic computers. A scheme for such a machine—an RCM (for relay computing machine, as distinguished from the now common ECM for electronic computing machine)—was adopted.

If the machine had been constructed quickly, it would have been our first high-speed digital computer. Incidentally, its computing speed exceeded that of the American ECMs of that time, due to a number of deep ideas implemented in its design. In particular, the 'cascade method' was used here, which is a kind of parallelization (a very pressing problem today). Also used in it was a 'Shannon counter' (which none of us had yet heard of), and similar devices.

All this opened completely new perspectives and revolutionized ideas about the possibilities of computing methods.

At the end of the 1940s the need was seen to complement the Kurchatov Institute with another 'atomic' institute, to be led by A. I. Alikhanov. At the recommendation of I. V. Kurchatov and L. D. Landau, Alikhanov invited Kronrod in 1949 to head the mathematics section of his institute, which came later to be called the Institute of Theoretical and Experimental Physics (ITEP). Here we should mention another feature of Kronrod's nature. He was a born organizer. Having been named head of the section, he had the opportunity to organize the work in it so that it was as productive as possible.

Computational mathematics, the machine, the possibility of organizing the work in his own way, recognition of the usefulness of this work—all these factors outweighed pure mathematics for Kronrod, who was, among other things, to a high degree a pragmatist.

When he went to the ITEP, Kronrod persuaded Bessonov to come along. The RCM was being built, but the construction was agonizingly slow. It was cheap, and this, unfortunately, caused it not to be taken very seriously. Quite well-meaning and competent people offered wise advice about how to speed up the construction by, for example, making the contacts out of gold: this would somewhat improve the quality of the machine, and it would correspondingly make it more expensive, which might radically change the attitude towards it. Kronrod could only be amused at such advice: his integrity would never allow him to resort to such tricks. And the years passed. By the time the RCM had been constructed, we had already begun designing the first ECMs. Thanks to the wealth of ideas put into the RCM, it still left the projected ECMs behind with regard to speed, but there was no doubt that it was not the future. Nevertheless, if the machine had been produced in good time (even with gold contacts), it would undoubtedly have paid for itself.

We have discussed the RCM at such length in order to underscore one of the principles to which Kronrod adhered strictly. Namely, the principle that the idea is nothing, its realization is everything. He was a person bursting with brilliant ideas, without placing the least value on them. He gave them out to right and left, and with complete sincerity and conviction regarded the person who realized his idea as its author. In this Kronrod was the total opposite of his teacher Luzin.

When speaking of the RCM, he firmly attributed its creation solely to Bessonov (who was, however, certainly a talented individual).

Having a clear and unprejudiced view, Kronrod quickly came to appreciate the advantages of electronic elements over relay elements. He took an active part in discussing the construction of the first ECMs as a member of the councils, so numerous and diverse in those years, for planning the creation of the machines. It should be mentioned that in those discussions he was often left in the minority, his ideas being ahead of their time. For example, he unsuccessfully argued that operations with floating point numbers in the machine should be realized in the hardware. Nevertheless, our first machines had fixed point, and operations with floating point numbers were implemented through programs; formally, this made the speed of the machine greater, but in reality it reduced the speed to a very low level. All this aside though, the RCM remained his favourite brainchild, and there were tears in his eyes when it was dismantled.

In the period from 1950 to 1955 his main activity involved the numerical solution of problems in physics. He did much work with physicists, in particular, with theoretical physicists, among whom he was most closely connected with I. Ya. Pomeranchuk on a professional level, and on a purely human and friendly level with Landau. For solving problems of state importance he was awarded the Stalin Prize and the Order of the Red Banner of Labour.

Only in 1955 did Kronrod get a real chance to work with an ECM. This was the machine M-2, built by I. S. Bruk, M. A. Kartsev, and N. Ya. Matyukhin at the Krzhizhanovskii Institute of Energy, in a laboratory headed by Bruk which later grew into the Institute of Electronic Controlling Machines. The methods for communication between a mathematician and the machine were created by A. L. Brudno, a close personal friend of Kronrod and a like-minded thinker.

Here began a new period of Kronrod's life. This will be discussed later, but in order not to digress too much from the chronology we now mention another sphere of his activities. In the period 1946–1953 Kronrod led a seminar, called the 'Kronrod circle', which was probably no less known among the mathematical youth of that time than was Luzin's seminar in its time. An atmosphere of elation, enthusiasm, and conviction that there was no science but mathematics, with Kronrod as its prophet, prevailed in this circle. Yet at the same time Kronrod was not the master, but simply 'Sasha', as he remained Sasha to the end of his days. The topics of the circle: the theory of functions of a real variable, set theory, and set-theoretic topology. And when Kronrod himself stopped doing pure mathematics, he continued to lead this circle and with the same zeal. He regarded (both then and in later years) the theory of functions of a real variable as most suitable for introducing students to creative activities, because one can get new and complex results in it from a small store of initial knowledge. Many mathematicians of the now older generation passed through this circle (E. M. Landis, A. Ya. Dubovitskii, E. V. Glivenko, R. A. Minlos, F. A. Berezin, A. A. Milyutin, A. G. Vitushkin, R. L. Dobrushin, N. N. Konstantinov, and many others).

When MSU moved to a new building, Kronrod stopped leading the seminar. Within a short time he revived it, but now on a new basis, computers.

After beginning to invest more time with programming on the M-2 computer, Kronrod quickly arrived at the opinion that computational problems were not at all the main thing for which a computer can be used. The principal thing was to teach the machine to think, that is, what is now called ‘artificial intelligence’, but what was then called ‘heuristic programming’.

Kronrod attracted a large group of mathematicians and physicists (Adel’son-Vel’skii, Brudno, M. M. Bongard, Landis, Konstantinov, and others). And while some of them arrived at similar problems independently, in any case they certainly acknowledged Kronrod’s leadership. A new ‘Kronrod circle’ began to function in the room next to the one containing the M-2 machine. The lively topics of discussion in this seminar centred on methods of pattern recognition (this work, headed by Bongard, culminated in the creation by his group of the well-known program ‘Cortex’, variants of which are still in operation), the transport problem (Brudno introduced this problem into the seminar and studied it intensively), problems in the theory of automata, and many other problems.

Kronrod knew how to direct the enthusiasm of the participants into a practical vein. He proposed choosing a standard problem such that progress in its solution would enable them to judge the level reached by the authors in the area of heuristic programming. He suggested an intellectual game as such a problem. A Russian card game (Podkidnoi Durak) was the first problem to be chosen and programmed. This choice (in spite of frequent smiles) was not random and was not dictated by the wish to be eccentric. It was a complicated game, not having a developed theory and yet admitting a simple description of position, which was extremely important because of the limitations of the computer in power and memory. A program was written and the game played. The game program was sufficiently powerful as long as there were still cards in the deck and the game proceeded ‘with incomplete information’. But the computer was not powerful enough after the game passed into the ‘open’ stage with the cards showing when everything reduced to a sorting of variants, because of the unusually large size of the ‘game tree’. (They never again returned to this game, and there is even some doubt as to whether modern computers are sufficiently powerful for it.)

In the process of creating this program the general principles needed for heuristic programming were first discovered: a process for sorting by the program that is not dependent on the length of the sorting (it was not a priori clear whether this was possible), algorithms for organizing information, and so on. But the game ‘Podkidnoi Durak’ obviously did not satisfy the requirements for a ‘standard’, since it was a purely regional (or national) game, and Kronrod proposed another game—chess—as a standard. Chess is played over the whole world. People in the USA had already begun to write chess programs. A chess program was now created on the basis of ‘proper’ computers: an M-20 computer was installed in the mathematics section of the ITEP, and later a second M-20. A chess program was written by a group of mathematicians (Adel’son-Vel’skii, V. L. Arlazarov, A. R. Bitman, and A. V. Uskov) without the participation of Kronrod himself, though he did involve himself when difficulties arose (connected with recursion) in creating a general sorting scheme: he thought of an improvement by which the difficulties could be overcome. Kronrod took upon himself the role of an organizer. It was necessary (and this was not simple) to create conditions for the chess group to work in an institute where by far the most researchers looked at heuristic programming, and in general at all that was not concerned with their immediate areas, as an overindulgence.

He arranged a chess match between the ITEP program and the best American chess program at that time, a program created by a group at Stanford University under the supervision of J. McCarthy. The match consisted of four games, played by telegraph, and ended 3 : 1 in favour of the ITEP program.

However, the mathematics section of the ITEP existed, of course, to help with physics problems, and the time has come to talk about how this work was organized by Kronrod. This can be instructive, because in all scientific research institutes there is a need for mathematical services, and they are not provided in the same way everywhere.

Kronrod considered that a mathematician solving the mathematical part of a physics problem had to understand this problem, starting from its formulation, and he also had to understand how the results obtained would be used. Then the mathematician worked out an algorithm, taking into account the physical formulation as a rule, wrote a program, and computed. He had to program it himself because only thus was it possible to choose an optimal variant of solution. For all this a mathematician of sufficiently high qualification was required, and Kronrod attracted many good graduates of MechMat to the ITEP, including some who specialized in the most abstract areas. Why specifically MechMat? He liked to repeat Gel'fand's words: "The task of MechMat is to make people competent", understood in the sense that for such a mathematician it should suffice to formulate the definitions and the rules for operating with them.

But for a mathematician himself to be able to program without expending needless effort he had to be given a maximum of conveniences and to be freed from all work not requiring his qualifications. The mathematician wrote in a convenient language close to his usual language, on a form printed on good quality paper, and with a pencil, so that he could erase any number of times. At his disposal he had a rich library of standard programs which were conveniently accessible to him. The written program (or any segment of it) was sent to be coded. The coding, checking of the coding, card punching, checking of the punched cards—the programmer did not have to think about this. The next day he received two copies of the deck of program cards without a single error in the coding or the card punching. The debugging was done at the control panel, and there were no time problems. The programmer could sit at the control panel as many times as he wanted. And not very much was needed: the program was broken up into small blocks, each was debugged separately, and it usually ran 'with a push of the button'. A correction could be inserted in the program via the keyboard, as is now done with a text editor. A card puncher was on duty specifically for the debugging programmer, in order to punch the card immediately. Coloured punched cards were used for this purpose. On another day the corrected white card took its place in the deck.

Controls in the form of hand calculations were obligatory. The following rule was strictly observed: if a program worked and gave a likely result, that did not yet mean it worked correctly, even if the result was accurate in degenerate cases.

The coding and card punching work proved to be extremely important in the composition cycle of the program. Women did the work at that stage, since experience had shown that they could cope with this work better than men. Printed at the bottom of each of the program forms made for Kronrod's section were the following lines: "program composed by ...", "coded by ...", "coding checked by ...", "cards punched by ...", "card punching checked by ...".

How did Kronrod get such accurate work from these subsections? First, he chose good workers; second, he got high wages for them; and finally, he made the wages dependent on the quality of work. Error-free work was rewarded by a 20 % monthly bonus. If a person checking punched cards overlooked two errors in a month, then this bonus was reduced by half. Missing two additional errors in a month meant no bonus at all (errors in the coloured punched cards were not counted).

Here the kind and gentle Kronrod was inflexible. On the other hand, in any matter not concerned with the quality of their work he tried to meet them halfway whenever possible. They loved him, valued their work and considered it important, and there were no errors.

Bessonov quickly qualified himself as an expert in electronics, and kept the machines in exemplary working order. There were almost no failures. Here we should mention that under Kronrod's supervision Bessonov was constantly introducing improvements in the machines. In 1963 Bessonov completely altered the system of commands, thereby doubling the power of the machines.

Kronrod started out from the principle that a normal computational problem should be computed quickly. Of course, there were special cases when a long computation was necessary, but this was not the rule, and exceptions were fairly rare.

The following was the accepted procedure: if a debugged program ran longer than ten minutes, then its author was summoned before the 'council of elders', with Kronrod at its head. The algorithms were analyzed, and the computation time was usually reduced.

All this as a whole was like an efficient factory production line, and the results were striking. Using computers with little power for difficult problems, the ITEP mathematicians surpassed the West. For example, the program for processing scintillation chamber observations gave high accuracy and ran twice as fast on the M-20 than the analogous CERN program on a computer having 500 times the speed. In several hours of the night it could compute everything that the accelerator could give in 24 hours of operation. There was thus time also for debugging and for preventive maintenance of the machine (for vacuum-tube computers this was a necessity), and there was much time for heuristic and other problems, of which we shall speak later.

In the world of our theoretical physicists of that time a clear law was observed: the more talented the theoretical physicist, the less computing was done for him. There was one physicist for whom no computing at all was done: Landau. As a rule, the weaker physicists wanted much computing done for them, and some of them showed displeasure when mathematicians were interested in where they got the equation they had brought for solution or what they were going to do with the result obtained. Here it should be noted that Kronrod liked to cite Hamming: "Before you begin the calculations, think about what you will do with the results." In Kronrod's section it was understood that every problem that led to a large computation would be discussed with a mathematician, who would have to go into the physical formulation. Here it was sometimes revealed that a qualitative result was needed that could be found without computing, that the problem was overdetermined or underdetermined, that the computational errors exceeded the effect sought, that the problem was ill-posed, and so on. Kronrod even hung a sign on his office door: "Do not enter with integral equations of the first kind!" This did not at all mean that he regarded integral equations of the first kind as impossible to solve. For example, the shapes of the poles of the magnets for the ITEP's 10-GeV accelerator were computed in the mathematics section, and then also for the Serpukhov accelerator. The computation of the poles was a Cauchy problem for the Laplace equation, and was well known to reduce to an integral equation of the first kind. But this was a special case—it was indeed necessary to compute. Incidentally, this work came directly from the distinguished mathematician A. M. Il'in.

Returning to Kronrod, we must say that while he understood that there were cases when an equation of the first kind had to be solved as an essential part of the problem, he saw at the same time that it was much more frequent to encounter problems where the result of solving an equation of the first kind was not in itself needed—it was rather some average of the solution that was of interest, and a simpler and (the main thing) well-posed problem would usually suffice to get this averaged quantity.

Two and a half decades have passed since that time, and several generations of computers. Their speed has increased by many orders of magnitude, and their memory has become practically unlimited. The way in which people communicate with computers has changed, as has the way in which people use them. They are mainly used not for computations but for processing and storing information. And yet much of what was put into practice by Kronrod has retained its relevance up to the present time. If a mathematician is involved in the solution of a problem in the natural sciences (in the role of a computer or programmer), then he must begin the work by interpreting the physical, chemical, biological, economic, ... formulation of the problem. He must work together with the physicist, chemist, biologist, economist if it is a strong physicist, chemist, biologist, economist (or in place of him if not) to produce a mathematical formulation, and then devise an algorithm and write a program, while understanding that for a serious problem the reasonableness of the algorithms composed will become clear only in the process of writing the program. Here the mathematician should be given a maximum of conveniences and freed from all work not requiring his qualifications.

From the end of the 1950s Kronrod began to be interested in questions of economics, in particular, questions of price formation. He focused attention on the fact that the principles at the basis of price formation are fallacious. L. V. Kantorovich and a number of economists arrived at the same conclusions. The Commission on Price Formation was created, attached to the Council of Ministers, and Kantorovich and Kronrod were among the mathematicians on it. New principles of price formation were adopted as a result of the activity of this commission. To implement these principles it was necessary to compute the so-called 'Leont'ev matrices' for the balance of material expenditures in the country. This massive computational work was headed by Kronrod and was done first on the RCM, and then on the same two M-20 machines. The work was later continued by a student of Kronrod, the now well-known economist V. D. Belkin.

Another problem which interested Kronrod in the 1960s was the problem of differential diagnostics of certain illnesses with the help of a computer. At the

Hertzen Oncological Institute a laboratory was created and headed by P. E. Kunin, a physicist by education and a student of Kronrod in the area of heuristics. Research was done in this laboratory, in particular, on differential diagnostics of lung cancer and central pneumonia (which was extremely important, since recommendations to perform surgery depended on it). This work was headed by Kronrod. Very promising results were achieved, but the sudden death of Kunin interrupted the work.

At the same time Kronrod was organizing mathematics classes in middle school and developing teaching methods for these classes.

After working at the ITEP, Kronrod became in 1968 the head of the mathematics laboratory of the Central Scientific Research Institute of Patent Information (CSRIPI).[4] After putting in order the mathematics-informational part of his job (and to this end he needed, among other things, to create for the CSRIPI computer 'Razdan' some software meeting Kronrod standards and to put together an amicable group of mathematicians), he became interested in patent matters proper, and he discovered that radical reform was needed there in order to stimulate inventive activities.

Kronrod worked out a scheme for a number of measures that should have resulted in an improvement of the situation, and he took the scheme to higher authorities, where he found understanding. However, the director, who had supported him, was replaced, and the new director hurried to distance himself from such a disturbing employee.

Kronrod left the CSRIPI.

The last place he worked was an institute called the Central Geophysical Expedition. Here Kronrod was head of a laboratory for computing with instrument readings taken when drilling test wells. He implemented a number of new computational ideas, but this work did not, of course, correspond to the scale of his talents, and he found something new to do.

It should be noted that Kronrod attracted many talented people of the most diverse areas of specialization. And while his professional potentialities were what interested some of them (for example, he calculated optimal modes for recovering oil and gas deposits for the well-known oil industry official Lopuk), his relations with others were in the most varied circles of interests. At his home could be found the actor Evstigneev or the screenwriter Nusinov. Kronrod might be seen together with I. G. Petrovskii at an exhibition of Burdel's sculptures, discussing questions not at all concerned with mathematics but with the fine arts. Many of his acquaintances were also prominent physicians: the surgeon Simonyan, the pediatrician Pobedinskaya, the radiation oncologist Marmorshtein, and others.

Possessing a heightened feeling of love for his fellow-man and an aspiration to help those who suffered immediately, this very minute, Kronrod avidly absorbed the professional accounts of the physicians, vicariously experiencing their successes and failures, gradually coming to the conviction that to save a mortally ill human being was the most important thing that could and should be done. At the time he met the Bulgarian physician Bogdanov, creator of a preparation called anabol made

[4]He was fired from the ITEP in connection with his signing a letter in defence of A. S. Esenin-Vol'pin.

from deactivated *lactobacillus bulgaricus*. This preparation had frequently caused a lengthy remission in cancer patients. Among others, Bogdanov had treated I. N. Vekua and S. A. Lebedev with anabol.

Kronrod began to propagandize this preparation in every way possible. It was not very easy to get: it was produced in Bulgaria in limited quantities. Kronrod arranged its delivery for dying patients. But this was not the way out of the situation—the supply of anabol was small, and it was expensive. It was necessary to manufacture it in larger quantities and by simple means. He tried to think of such a way. Thus appeared a new preparation—yogurt made with the *bacillus bulgaricus*—which he called "milil" (in honour of Mechnikov, Il'ya Il'ich). He worked out a simple technology for producing it and also ways of using it.[5]

Kronrod did not treat patients without the help of physicians. Physicians used milil according to his instructions (and there were more and more physicians who believed in Kronrod's preparation), and only in hopeless cases—they treated patients known to be condemned to a quick death. Milil acquired a reputation and even a certain amount of recognition: in his institute A. A. Vishnevskii set aside a ward for the treatment of patients by Kronrod's method. Kronrod was promised a laboratory for experiments on animals. However, nothing came of the promises, and he performed all his experiments on himself.

Kronrod was no longer a novice at medicine. The mathematical books on his shelves were replaced by books on medicine, many obtained from the physicians he associated with. He already had considerable clinical experience. He had a huge card-index with histories of the illnesses of patients. And he had a large advantage over physicians: he could draw correct statistical conclusions from this card-index with its thousands of records. The prominent therapeutic physician I. G. Barenblatt (father of the specialist in mechanics G. I. Barenblatt) was struck by Kronrod's medical erudition after talking with him. But why should that be striking? If a very talented person makes an intense effort to study medicine and competent specialists help him in this, then he will probably learn it no worse than the average or even the good medical student. But he did not have a medical degree, and milil had not been introduced into pharmacology. Medicine could not tolerate this. We need only recall the story involving artificial pneumothorax. A criminal case was initiated against him, in spite of the fact that Kronrod did not adminster treatments except in the presence of a physician and did not take money for them but, on the contrary, spent all his own money on the treatments (reaching the point that his laboratory assistants gave him a suit for his birthday, his was so worn). Also—the main thing—his card-index was confiscated. This story ended tragically, or better said, tragicomically. Either the wife or mother of the prosecutor who had instituted the proceedings became ill with cancer, and he needed milil. The case was dropped, of course. The card-index was returned. But for Kronrod himself this turned into a tragedy. He suffered a stroke. He completely lost his speech. He forgot how to read and write. Recovery was slow. He again learned to speak, to read, to write. He quit his job at the Central Geophysical Expedition, and he made a final break with mathematics. Only medicine now interested him. But then he

[5] The editor of *Uspekhi Matematicheskikh Nauk* does not at present know of any data supporting the effectiveness of milil.

had a second stroke. The situation was hopeless. The physicians thought that the death agony was beginning. But he was conscious, and he asked to be put in a very hot bath and left there several hours. A prominent neuropathologist said later that it was the only right decision. This time he was able to save himself. He did not survive a third stroke.

He died on 6 October 1986.

*E. M. Landis and I. M. Yaglom*

### List of the main mathematical papers of A. S. Kronrod

[1] "On the structure of the set of points of discontinuity of a function differentiable at its points of continuity", *Izv. Akad. Nauk SSSR Ser. Mat.* **1939**, 569–578.

[2] "On the permutability of the terms of numerical series", *Dokl. Akad. Nauk SSSR* **49** (1945), 163–166.

[3] with G. M. Adel'son-Vel'skii, "On the level curves of continuous functions having partial derivatives", *Dokl. Akad. Nauk SSSR* **49** (1945), 239–241.

[4] with G. M. Adel'son-Vel'skii, "On the maximum principle for solutions of a system of elliptic partial differential equations", *Dokl. Akad. Nauk SSSR* **49** (1945), 559–561.

[5] with G. M. Adel'son-Vel'skii, "A direct proof of the analyticity of a monogenic function", *Dokl. Akad. Nauk SSSR* **50** (1945), 7–10.

[6] "On permutations of the terms of numerical series", *Mat. Sb.* **18 (60)** (1946), 237–280.

[7] with E. M. Landis, "On level sets of functions of several variables", *Dokl. Akad. Nauk SSSR* **58** (1947), 1269–1272.

[8] "Bounded variation and a total differential for functions of several variables", *Uspekhi Mat. Nauk* **2**:2 (18) (1947), 192–193.

[9] with E. M. Landis, "On smoothness of surfaces of functions of several variables", *Uspekhi Mat. Nauk* **2**:3 (19) (1947), 176.

[10] "On tangent planes to surfaces", *Uspekhi Mat. Nauk* **2**:4 (20) (1947), 167–168.

[11] "The integral and derivative for functions of several variables", *Uspekhi Mat. Nauk* **3**:2 (24) (1948), 220–222.

[12] "On surfaces of bounded area", *Uspekhi Mat. Nauk* **4**:5 (33) (1949), 181–182.

[13] "On the linear and the planar variations of functions of several variables", *Dokl. Akad. Nauk SSSR* **66** (1949), 797–800.

[14] "On a line integral", *Dokl. Akad. Nauk SSSR* **66** (1949), 1041–1044.

[15] "On functions of two variables", *Uspekhi Mat. Nauk* **5**:1 (35) (1950), 24–134.

### On the authors

E. M. Landis (6 October 1921–12 December 1998) was a prominent Soviet mathematician, a student and friend of Kronrod. Landis went through the Finnish War and World War II, from the first to the last day. He returned to resume his first year at MechMat in 1946 when Kronrod, of the same age but demobilized in 1943, had already graduated. The first papers of Landis were completed under the influence of Kronrod, and sometimes jointly with him ([7], [9]). Later Landis became a student of I. G. Petrovskii. This determined the main area of his mathematical research: partial differential equations. Landis worked for more than forty years in the Mechanics and Mathematics Department of MSU, where he was one of the leading professors. At the same time, he conducted applied research up to 1968 in a section of the Institute of Theoretical and Experimental Physics; the section was headed by Kronrod. Landis and Kronrod remained friends throughout their whole lives.

Isaak Moiseevich Yaglom was a well-known geometer and popularizer of science. He was born on 6 March 1921 in Khar'kov. In 1925 his family moved to Moscow. In school Yaglom was very attracted by mathematics. He studied in Shklyarskii's circle. In the fall of 1938 he was awarded a first prize in the Moscow mathematical olympiad. After finishing school he enrolled in MechMat, started attending Professor V. F. Kagan's seminar, and began to study under his supervision. At the beginning of the war he was evacuated to Sverdlovsk and entered Sverdlovsk University for his fourth year (in the war years the last), which ended in June 1942. In 1945 he defended his Ph.D. dissertation.

After graduate school Yaglom worked for two years in the mathematics editorial office of the Publishing House of Foreign Literature, and then for two years at MechMat. After these two years he could not find a job anywhere, for understandable reasons—this was the period from the end of the 1940s to the beginning of the 1950s. Then he worked at the Orekhovo Zuevo Pedagogical Institute. At the time of the Khrushchev thaw (in 1957) he moved to a job at the Lenin Pedagogical Institute, in the geometry section. In 1968 he lost his job because he had signed the letter in defence of Esenin-Vol'pin. He was accepted for a job at Yaroslavl University. In 1988 he died.

Yaglom was the author of about 25 popular books on mathematics for schoolchildren and students. He received the International Prize of the USA for Popularization of Science. (This prize is awarded for the popularization of all sciences, and Yaglom was the first to receive it for mathematics.)

This article was written soon after Kronrod's death, but was sent to the Russian editor only recently. The text has been published without changes.

Translated by H. H. McFADEN

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