I. M. Krichever, Obstructions to the existence of S^1-actions. Bordism of ramified coverings, *Mathematics of the USSR-Izvestiya*, 1976, Volume 10, Issue 4, 783–797

DOI: 10.1070/IM1976v010n04ABEH001814

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Ser. Mat. Tom 40 (1976), No. 4

Math. USSR Izvestija
Vol. 10 (1976), No. 4

OBSTRUCTIONS TO THE EXISTENCE OF S^1-ACTIONS. BORDISM OF RAMIFIED COVERINGS

UDC 513.83

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Abstract. In this article the author proves that the values of the multiplicative genera A_k under discussion, where $K = 2, 3, \ldots$, are obstructions to the existence of nontrivial S^1-actions on a unitary manifold whose first Chern class is divisible by k. The effective computation of these obstructions is carried out for algebraic manifolds. Simultaneously, formulas for the bordism class of a ramified covering are obtained.

Bibliography: 8 titles.

This article consists of two parts which look quite heterogeneous at first glance; nevertheless, they are connected by a unique approach. In the first part, the results of [1] receive further development. We recall that in [1], for each Hirzebruch genus h: $U_* \longrightarrow Q$, an invariant analog was constructed: a homomorphism h^G: $U_*^G \longrightarrow K(BG) \otimes Q$ from the bordism ring of manifolds with actions of a compact Lie group G into the rational K-functor of the universal classifying space BG. The study of "equivariant Hirzebruch genera" permitted us to express the value of h on the bordism class of a G-manifold through invariants of its fixed submanifolds. The little-effective cumbersome formulas for the general case became extremely simple for the symmetric analog of the classical genus T_y: the two-parameter genus $T_{x,y}$ whose value on the bordism class of complex projective space CP^n is equal to $\Sigma_{i=0}^n x^i y^{n-i}$. (Note that this definition of $T_{x,y}$ differs from that given in [1] by substitution of $-y$ for y.)

In this article, for the Hirzebruch genera A_k, $k = 2, 3, \ldots$, given by the series $kte^t/(e^{kt} - 1)$ we shall prove the following result:

THEOREM 2.2. *If the action of a compact connected Lie group G on a manifold X whose tangent bundle has first Chern class $c_1(X) \in H^2(X, Z)$ divisible by k is nontrivial, then*

$$A_k^G([X, G]) = 0;$$

in particular, $A_k([X]) = 0$.

The proof of this theorem, as the proof of the results of [1], is based on arguments

AMS (MOS) subject classifications (1970). Primary 55G35, 57D15, 55A10; Secondary 57E25, 57D85.

on the analyticity of functions connected with equivariant genera. To explain the fundamental idea, consider an S^1-manifold X with isolated fixed points x_s. Assume that the representation of S^1 in the fibers of the tangent bundle over x_s is $\Sigma_{i=1}^n \eta^{j_{si}}$, where η^j is the jth power of the standard one-dimensional representation of S^1; then $A_k^{S^1}([X, S^1]) \in K(CP^\infty) \otimes Q = Q[[\eta - 1]]$ coincides with the Laurent series expansion in a deleted neighborhood of 1 of the rational function

$$\mathfrak{A}(\eta) = \sum_s \prod_{i=1}^n \frac{k\eta^{j_{si}}}{\eta^{kj_{si}} - 1}.$$

Therefore $\mathfrak{A}(\eta)$ does not have a pole at 1. It turns out that if $c_1(X)$ is divisible by k, then $\mathfrak{A}(\eta)$ is analytic at all the roots of 1, hence everywhere. Since $\mathfrak{A}(0) = \mathfrak{A}(\infty) = 0$, it follows that $\mathfrak{A}(\eta) \equiv 0$ and, in particular, $A_k([X]) = 0$.

As is known, an algebraic manifold X given in CP^n by a system of homogeneous polynomials $P_{m_i}(x_0, \ldots, x_n)$ is uniquely determined up to diffeomorphism by the powers m_i, $1 \leqslant i \leqslant s$. In §3 we obtain explicit formulas which reconstruct the bordism class of X starting from the numbers m_i (see also [2]) and, in consequence, also its multiplicative genera. Together with Theorem 2.2, these formulas prove the following theorem.

THEOREM A. *If the coefficient of $(t-1)^n$ in the expansion (in powers of $t - 1$) of the function $t^{-2}\Pi_{i=1}^s(t^{m_i} - 1)$ is different from zero, then there exist no nontrivial actions of S^1 on X.*

The methods we develop allow us in §4 to pass to the second part of the paper and to solve the problem of reconstructing the bordism class of a ramified covering. More precisely: the projection $p\colon Y \longrightarrow X$ of a ramified n-fold covering of X with ramification along a submanifold F determines a bordism class $[Y, p] \in U_*(X) \otimes Q$. If $v \in U^2(X)$ is the cobordism class dual to F in X, then we have

THEOREM 4.1. *The cobordism class dual to $[Y, p]$ is equal to*

$$\frac{v}{g^{-1}(n^{-1}g(v))} \in U^0(X) \otimes Q,$$

where

$$g(t) = \sum_{n=0}^\infty \frac{[CP^n]}{n+1} t^{n+1}.$$

The formulas for genera of a ramified covering and, in particular, the signature formula of Hirzebruch [3] are a simple consequence of this theorem.

It should be noted that although most results carry over to other theories practically without changes, for the sake of definiteness we assume that manifolds, group actions on them and bundles are unitary, unless otherwise stated.

§1. Basic definitions and necessary information

As we have already said, this work continues the research started in [1]. To make the presentation of results self-contained as far as possible, all the necessary information contained in [1] is gathered in this section.

1. Two collections $\Xi^{(1)} = \{\xi_i^{(1)}\}$ and $\Xi^{(2)} = \{\xi_i^{(2)}\}$, $1 \leqslant i \leqslant r$, of G-bundles over G-manifolds X_1 and X_2 respectively are said to be bordant if there is a collection $Z = \{\zeta_i\}$ of G-bundles over a G-manifold W whose boundary is isomorphic with $X_1 \cup -X_2$ such that the restriction of ζ_i to $X^{(j)}$, $j = 0, 1$, is isomorphic with $\zeta_i^{(j)}$. In the usual way, the disjoint union of collections of G-bundles turns the set of classes of bordant collections such that the real dimension of their basis is n and $\dim_C \xi_i = \mu_i$ into a group $U^G_{n,\mu}$, $\mu = (\mu_1, \ldots, \mu_r)$. The subgroups $U^G_{n,0}$ are naturally identified with bordism groups of G-manifolds.

As fundamental invariants of a collection of G-bundles we have the values taken on its bordism class by "equivariant characteristic homomorphisms"

$$\chi^G : U^G_{n,\mu} \to U^{-n+k}(BG),$$

defined for each characteristic class in the cobordism of collections of vector bundles $\chi \in U^k(\prod_{i=1}^r BU(\mu_i))$.

For each G-space X, we denote by X_G the set $(X \times EG)/G$. Analogously, for a G-bundle ξ over X we define a vector bundle ξ_G over X_G. Then to a collection Ξ of G-bundles there corresponds a collection Ξ_G of bundles over X_G. The value of χ^G on a bordism class $[\Xi] \in U^G_{n,\mu}$ is given by

$$\chi^G(\Xi) = p_!(\chi(\Xi_G)),$$

where $p_!$: $U^*(X_G) \to U^{*-n}(BG)$ is the Gysin homomorphism induced by the projection p: $X_G \to BG$. In what follows, the homomorphism $U^G_* \to U^{-*}(BG)$ corresponding to the characteristic class $1 \in U^0$ will be denoted by χ^G_0.

Consider an arbitrary G-manifold X. As it is known (see [4]), there exists an equivariant embedding of X into some G-module $\widetilde{\Delta}$. We denote by Δ the maximal direct summand of $\widetilde{\Delta}$ whose restriction to a normal factor H of G does not contain trivial summands.

THEOREM 1.1. *Assume that a collection Ξ_s of G-bundles is obtained by restriction of G-bundles Ξ to a G-submanifold F_s which is fixed under the action of H. Then*

$$e(\Delta_G)\,\chi^G(\Xi) = \sum_s p_{s!}[e((-\nu_s)_G)\,\chi(\Xi_{sG})],$$

where $p_{s!}$ is the Gysin homomorphism induced by the projection p_s: $F_{sG} \to BG$, $(-\nu_s)$ is the G-bundle over F_s whose sum with the normal G-bundle ν_s of F_s in X is equal to the G-bundle $\Delta \times F_s \to F_s$ and $e(\ )$ is the Euler class of a bundle.

To solve the equation in $\chi^G(\Xi)$ given by Theorem 1.1 in the case when H coincides with G (this will be assumed throughout this section), we introduce the localization $U^*(BG)_{\mathfrak{S}}$ of the ring $U^*(BG)$ at the multiplicative set $\mathfrak{S}$ of Euler classes of bundles associated with representations Δ of G having no trivial summands.

We denote by $\bar{\chi}^G$ the composite

$$U^G_{*,*} \xrightarrow{\chi^G} U^*(BG) \to U^*(BG)_{\mathfrak{S}}.$$

By Theorem 1.1,

$$\overline{\chi}^G(\Xi) = \sum_s \frac{1}{e(\Delta)} p_{s!}\,[e((-\nu_s)_G)\,\chi(\Xi_{sG})].$$

Since $\nu_s + (-\nu_s)$ is the G-bundle $\Delta \times F_s \longrightarrow F_s$, we have

$$p_s^*(e(\Delta_G)) = e((\nu_s + (-\nu_s))_G) = e(\nu_{sG})\,e((-\nu_s)_G).$$

Therefore

$$\overline{\chi}^G(\Xi) = \sum_s p_{s!}\left(\frac{\chi(\Xi_{sG})}{e(\nu_{sG})}\right). \tag{1.1}$$

To interpret correctly each term in the right side of this equality, we shall show that for a G-bundle ζ over a trivial G-manifold F having no trivial summands in the representation of G in the fibers and for every class $x \in U^*(F \times BG)$, the formula

$$p_!\left(\frac{x}{e(\zeta_G)}\right) \tag{1.2}$$

determines a class in $U^*(BG)_{\mathfrak{S}}$.

As is known, for a G-bundle ζ we have the decomposition $\zeta = \bigoplus_j \operatorname{Hom}_G(\Delta_j, \zeta) \otimes \Delta_j$, where the summation is over the nontrivial irreducible representations Δ_j of G. Let $\kappa_j = \operatorname{Hom}_G(\Delta_j, \zeta)$; then $e(\zeta_G) = \prod_j e(\kappa_j \otimes \Delta_{jG})$.

We shall find $e(\kappa \otimes \Delta_G)$ for an arbitrary bundle κ over F and a representation Δ of G:

$$e(\varkappa \otimes \Delta_G) = \prod_{m,l}\left(\lambda_m + \rho_l + \sum_{i,j>1} \alpha_{ij}\lambda_m^i\rho_l^j\right),$$

where the λ_m and ρ_l are the Wu generators of $\kappa \otimes 1$ and $1 \otimes \Delta_G = p^*(\Delta_G)$ respectively; $f(u, v) = u + v + \Sigma_{i,j>1}\, \alpha_{ij}u^iv^j$ is the formal group of "geometric cobordisms". We recall that to each homogeneous symmetric polynomial P_ω of degree n in the Wu variables of a bundle, there corresponds a characteristic class of degree n. Thus,

$$e(\varkappa \otimes \Delta_G) = p^*(e(\Delta_G)) + \sum_{\omega,\omega'} \beta_{\omega\omega'} P_\omega(\ldots, \lambda_m, \ldots) P_{\omega'}(\ldots, \rho_l, \ldots).$$

We denote by $\sigma(\kappa, \Delta)$ the sum in the right side of the equality. It is important to note that the degrees of all characteristic classes $P_\omega(\ldots, \lambda_m, \ldots)$ are greater than zero. Therefore the series

$$\frac{1}{e(\varkappa\otimes\Delta_G)} = \frac{1}{p^*(e(\Delta_G))}\left[\sum_{l=0}^{\infty} (-1)^l \left(\frac{\sigma(\varkappa, \Delta)}{p^*(e(\Delta_G))}\right)^l\right]$$

contains only a finite number of summands different from zero. This already makes it evident that expression (1.2) is correct.

2. Consider the category of pairs consisting of a G-bundle ζ_0 over a trivial G-manifold such that the representation of G in its fibers does not have trivial summands and a collection $Z = \{\zeta_i\}$ of G-bundles, $1 \leqslant i \leqslant r$, over the same base. We shall denote by $R^G_{n,\mu}$ the

bordism groups in this category, where n is the real dimension of the total space of ζ_0 and $\mu = (\mu_1, \ldots, \mu_r)$, where $\mu_i = \dim_{\mathbf{C}} \zeta_i$.

In the notation of the preceding subsection, for each fixed submanifold F_s of a G-manifold X, the pair (ν_s, Ξ_s) gives a bordism class belonging to $R^G_{*,*}$. Their sum determines the image $[\Xi] \in U^G_{*,*}$ under the homomorphism of bigraded rings

$$\beta^G : U^G_{*,*} \to R^G_{*,*}.$$

Now (1.1) leads us to the corollary:

COROLLARY. *For each characteristic class $\chi \in U^k(\Pi_{i=1}^r BU(\mu_i))$, the homomorphism*

$$X^G : R^G_{**} \to U^*(BG)_{\mathscr{G}},$$

given on (ζ_0, Z) by the formula

$$X^G(\zeta_0, Z) = p_1\left(\frac{\chi(Z_G)}{e(\zeta_{0G})}\right),$$

satisfies the equation $\bar{\chi}^G = X^G \circ \beta^G$.

For the group S^1, all the irreducible representations $\eta^j, j = 0, \pm 1, \ldots$, are obtained by tensoring the standard one-dimensional representation η by itself j times. In addition,

$$e((\eta^m)_{S^1}) = g^{-1}(mg(u)) = [u]_m \in U^*(CP^\infty),$$

where $g(u)$, the logarithm of the formal group of "geometric cobordisms", is equal to $\Sigma_{n=0}^\infty ([CP^n]/(n+1))u^{n+1}$. The ring $U^*(CP^\infty)_{\mathscr{G}}$ is isomorphic with $U^*[[u]] \otimes Q[u^{-1}]$.

Assume that ζ_0 and the ζ_i are decomposed in sums

$$\zeta_0 = \sum_l \varkappa_{j_l} \otimes \eta^{j_l}, \quad \zeta_i = \sum_m \varkappa_{j_{mi}} \otimes \eta^{j_{mi}}.$$

We denote by $\lambda^s_{j_l}$ and $\lambda^t_{j_{mi}}$ the Wu generators of the bundles κ_{j_l} and $\kappa_{j_{mi}}$ respectively. Then, if the characteristic class χ is given by a product of symmetric polynomials $P_i(x_1, \ldots, x_{\mu_i})$, we have

LEMMA 1.1. *The value of X^G on the bordism class of the pair (ζ_0, Z) is equal to*

$$p_1\left(\frac{\prod_i P_i(\ldots, f([u]_{j_{mi}}, \lambda^t_{j_{mi}}), \ldots)}{\prod_{l,s} f([u]_{j_l}, \lambda^s_{j_l})}\right).$$

3. Let us consider the functorial properties of the homomorphisms χ^G with respect to the group homomorphisms $\alpha: G_i \to G$. By means of α, each G-bundle is naturally transformed into a G_i-bundle. This transformation defines a transfer homomorphism:

$$\alpha^* : U^G_{*,*} \to U^{G_i}_{*,*}.$$

THEOREM 1.2. *For every characteristic class χ, the diagram*

$$\begin{array}{ccc} U^G_{*,*} & \xrightarrow{\chi^G} & U^*(BG) \\ a^\# \downarrow & & \downarrow a^* \\ U^{G_1}_{*,*} & \xrightarrow{\chi^{G_1}} & U^*(BG_1) \end{array}$$

is commutative.

4. By a theorem of Dold, to each Hirzebruch genus, i.e. homomorphism $h: U_* \longrightarrow Q$, there corresponds a transformation of functors $\widetilde{h}: U^*(\ ) \longrightarrow K(\ ) \otimes Q$ such that h coincides with the composite $U_* \xrightarrow{\sim} U^* \xrightarrow{\widetilde{h}} Q$.

LEMMA 1.2. *The value of $\widetilde{h}$ on the generator $u \in U^2(CP^\infty)$ is equal to*

$$\widetilde{h}(u) = g_h^{-1}(\ln \eta) \in Q[[\eta - 1]] = K(CP^\infty) \otimes Q,$$

where g_h^{-1} is the functional inverse to the series

$$g_h(t) = \sum_{n=0}^{\infty} \frac{h([CP^n])}{n+1} t^{n+1}.$$

DEFINITION. The *equivariant Hirzebruch genus* corresponding to a rational genus h: $U_* \longrightarrow Q$ is the homomorphism

$$h^G = \widetilde{h} \circ \chi_0^G : U^G_{eq} \to K(BG) \otimes Q.$$

For the trivial group $\{e\}$ consisting only of the identity, $h^{\{e\}}$ assigns to each G-manifold X the number $h([X])$; therefore from Theorem 1.2 we obtain the following proposition.

LEMMA 1.3. *The value of a genus h on the bordism class of a G-manifold X is equal to $\epsilon(h^G([X, G]))$, where ϵ: $K(BG) \otimes Q \longrightarrow Q$ is the augmentation.*

§2. Obstructions to the existence of S^1-actions

We define the rational genera A_k, $k = 2, 3, \ldots$, as the series $kte^t/(e^{kt} - 1)$.

From the viewpoint of the theory of characteristic classes, the value of a genus given by the series $t/h(t)$, $h(t) = t + \Sigma_{i=2}^{\infty} q_i t^i$, $q_i \in Q$, on the bordism class $[CP^n]$ is equal to the coefficient of t^n in the series $(t/h(t))^{n+1}$. As was proved in [6], we have $h(t) = g_h^{-1}(t)$.

THEOREM 2.1. *If for an S^1-bundle ξ over X the difference $c_1(X) - c_1(\xi)$ is divisible by k in $H^2(X, Z)$, then the series $A_k(p_!(e(\xi_{S^1}))) \in Q[[\eta - 1]]$ coincides with the expansion about 1 of some polynomial in the variables η and η^{-1}.*

PROOF. Assume that E^{S^1}: $R^{S^1}_{n,l} \longrightarrow U^{-n+l}(CP^\infty)$ is the homomorphism defined by the condition in the corollary of Theorem 1.1 in the case when the characteristic class is e, the Euler characteristic. If $\lambda^s_{j_l}$ and $\lambda^t_{j_m}$ are Wu generators of bundles κ_{j_l} and κ_{j_m} such that $\zeta_0 = \Sigma_l \kappa_{j_l} \otimes \eta^{j_l}$ and $\zeta = \Sigma_m \kappa_{j_m} \otimes \eta^{j_m}$, then, by Lemma 1.1, for the bordism class $[\zeta_0, \zeta] \in R^{S^1}_{*,*}$ we have

$$\widetilde{A}_k \circ \mathrm{E}^{S^1}([\zeta_0, \zeta]) = \widetilde{A}_k \circ p_!\left(\frac{\prod_m \prod_t f([u]_{j_m}, \lambda^t_{j_m})}{\prod_l \prod_s f([u]_{j_l}, \lambda^s_{j_l})} \right).$$

To find the series in $\eta - 1$ that appears in the right side of the equality, we must apply the homomorphism A_k to the coefficients of the formal group $f(u, v)$ which thus becomes $f_{A_k}(u, v)$; replace u by $\widetilde{A}_k(u)$. Under the composite $\widetilde{A}_k \circ \rho_1$, the symmetric polynomials in $\lambda^t_{j_m}$ and $\lambda^s_{j_l}$ go to rational numbers which may be different from zero only if the degree of the polynomials is not greater than the dimension of the F-basis of ζ_0 and ζ.

Since $g^{-1}_{A_k}(t) = (e^{kt} - 1)/ke^t$, we have

$$f_{A_k}(u, v) = g^{-1}_{A_k}(g_{A_k}(u) + g_{A_k}(v)) = \frac{(1 + r_k(u))^k (1 + r_k(v))^k - 1}{k(1 + r_k(u))(1 + r_k(v))},$$

where we have introduced the notation $r_k(t) = e^{g_{A_k}(t)} - 1$.

By Lemma 1.2,

$$\widetilde{A}_k([u]_j) = g^{-1}_{A_k}(jg_{A_k}(\widetilde{A}(u))) = g^{-1}_{A_k}(j \ln \eta) = \frac{\eta^{kj} - 1}{k\eta^j}.$$

The series $r(u)$ turns into

$$e^{g_{A_k}(g^{-1}_{A_k}(\ln \eta))} - 1 = \eta - 1.$$

Thus the series $\widetilde{A}_k \circ \mathrm{E}^{S^1}([\zeta_0, \zeta])$ is obtained from

$$\prod_{m,t} \frac{\eta^{kj_m}(1 + r_k(\lambda^t_{j_m}))^k - 1}{k\eta^{j_m}(1 + r_k(\lambda^t_{j_m}))} \cdot \prod_{l,s} \frac{k\eta^{j_l}(1 + r_k(\lambda^s_{j_l}))}{\eta^{kj_l} - 1 + \eta^{kj_l}\widetilde{r}(\lambda^s_{j_l})}$$

$$= \prod_{m,t} \frac{\eta^{kj_m}(1 + r_k(\lambda^t_{j_m}))^k - 1}{k\eta^{j_m}(1 + r_k(\lambda^t_{j_m}))} \cdot \prod_{l,s} \left[\frac{k\eta^{j_l}(1 + r_k(\lambda^s_{j_l}))}{\eta^{kj_l} - 1} \times \sum_{i=0}^{\dim F} (-1)^i \left(\frac{\eta^{kj_l}}{\eta^{kj_l} - 1} \right)^i \widetilde{r}^i_k(\lambda^s_{j_l}) \right]$$

by substituting some rational numbers for symmetric polynomials in $\lambda^t_{j_m}$ and $\lambda^s_{j_l}$. Here $\widetilde{r}_k(t) = (1 + r_k(t))^k - 1$. This implies

LEMMA 2.1. *The series* $\widetilde{A}_k \circ \mathrm{E}^{S^1}([\zeta_0, \zeta])$ *coincides with the expansion about* 1 *of the function*

$$\frac{\prod_l \eta^{j_l}}{\prod_m \eta^{j_m}} R(\eta^k),$$

where $R(\eta)$ *is a rational function having poles at the* j_l*th roots of* 1.

Therefore $A_k \circ p_!(e(\xi_{S^1}))$ has the form

$$\sum_s \frac{\prod_l \eta^{j_{ls}}}{\prod_m \eta^{j_{ms}}} R_s(\eta^k). \tag{2.1}$$

Here $\Sigma_l \eta^{j_{ls}}$ and $\Sigma_m \eta^{j_{ms}}$ are the decompositions into irreducible representations of the representations of S^1 in the fibers of the normal bundle over the fixed submanifold F_s and the restriction of ξ to F_s respectively.

We denote by $\mathfrak{A}(\eta)$ the rational function (2.1). Since $\widetilde{A}_k \circ p_!(e(\xi_{S^1})) \in Q[[\eta]]$, $\mathfrak{A}(\eta)$ does not have poles at 1. Our immediate task will be to prove that there are no poles at the roots of 1. To this end we shall use Theorem 1.1. Below, F_s will denote a fixed submanifold of the action of a cyclic subgroup of order n of S^1. In the notation of Theorem 1.1,

$$\widetilde{A}_k \circ p_!(e(\xi_{S^1})) = \prod_d \frac{k\eta^{j_d}}{\eta^{kj_d}-1} \cdot \sum_s \widetilde{A}_k \circ p_{s!}[e(((-\nu)+\xi_s)_{S^1})]. \tag{2.2}$$

The first factor on the right side of this equality is $1/\widetilde{A}_k(e(\Delta_{S^1}))$. Since $\Delta = \Sigma_d \eta^{j_d}$ does not contain trivial representations of Z_n, the j_d are not divisible by n.

Now consider an arbitrary S^1-bundle ζ over an S^1-manifold F such that the action of the subgroup Z_n is trivial on it. ζ can be represented as a sum of S^1-bundles ζ_r, $0 \leqslant r \leqslant n - 1$. The generator of Z_n in the fibers of ζ_r acts by multiplication by $\exp(2\pi i r/n)$; therefore its action on $\widetilde{\zeta}_r = \zeta_r \otimes \eta^{-r}$ is trivial. Since $\zeta_r = \widetilde{\zeta}_r \otimes \eta^r$, we have

$$\widetilde{A}_k \circ p_!(e(\zeta_{S^1})) = \widetilde{A}_k \circ p_!\Big(\prod_r e(\widetilde{\zeta}_r \otimes \eta^r)\Big).$$

If Λ_r^m denotes the Wu generators of $\widetilde{\zeta}_r$, then

$$\widetilde{A}_k \circ p_!(e(\zeta_{S^1})) = \widetilde{A}_k \circ p_!\Big(\prod_{r,m} f(\Lambda_r^m, [u]_r)\Big)$$

$$= \widetilde{A}_k \circ p_!\left[\prod_{m,r} \frac{(1+r_k(\Lambda_r^m))^k \eta^{kr} - 1}{k\eta^r(1+r_k(\Lambda_r^m))}\right] = \widetilde{A}_k \circ p_!\left[\prod_r \frac{\sum_{i=0}^{\mu_r} \eta^{kri} P_i(\ldots, \Lambda_r^m, \ldots)}{\prod_m \eta^r(1+r_k(\Lambda_r^m))}\right],$$

where $\mu_r = \dim_{\mathbf{C}} \widetilde{\zeta}_r$ and $P_i(\ldots, \Lambda_r^m, \ldots)$ is the symmetric series. We denote by $\rho_i \in U^*(BU(\mu_r))$ the characteristic class defined by the symmetric series

$$\frac{P_i(x_1, \ldots, x_{\mu_r})}{\prod_m (1+r_k(x_m))},$$

and by ρ_ω we denote the product $\Pi_{r=0}^{n-1} \rho_{i_r} \in U^*(\Pi_{r=0}^{n-1} BU(\mu_r))$, $\omega = (i_0, \ldots, i_{n-1})$, $0 \leqslant i_r \leqslant \mu_r$. This characteristic class gives the homomorphism

$$\mathrm{P}_\omega^{S^1} : R_{*,\mu}^{S^1} \to U^*(CP^\infty)_{\mathscr{C}}.$$

As in the proof of Lemma 2.1, we use the corollary of Theorem 1.1 to obtain the following statement immediately from the above formulas:

LEMMA 2.2. *The series $A_k \circ P_\omega([\zeta_0, Z])$ coincides with the expansion about 1 of the function*

$$\frac{\prod_l \eta^{j_l}}{\prod_{m,r} \eta^{j_{mr}}} \cdot M_\omega(\eta^k), \tag{2.3}$$

where $M_\omega(\eta)$ is a rational function with poles at the j_lth roots of 1.

Then $\widetilde{A}_k \circ p_!(e(\zeta_{S^1}))$ coincides with the expansion about 1 of the rational function

$$\sum_\omega \prod_r \eta^{kri_r - r\mu_r} Q_\omega(\eta), \tag{2.4}$$

where the rational function $Q_\omega(\eta)$ has the form of a sum of functions (2.3) over all the fixed submanifolds of the S^1-manifold F. Its expansion about 1 coincides with the series $\widetilde{A}_k \circ \rho_\omega^{S^1}(\widetilde{\zeta}_r, \ldots, \widetilde{\zeta}_{n-1})$. Therefore $Q_\omega(\eta)$ does not have poles at 1. We shall prove that it does not have poles at the nth roots of 1.

In fact, the projection α of the group S^1 onto the factor group $S^1/Z_n = S^1$ induces a map α: $CP^\infty \longrightarrow CP^\infty$ such that $\alpha^*(\eta) = \eta^n$. Since the action of Z_n on each $\widetilde{\zeta}_r$ is trivial, $\widetilde{\zeta}_r$ is the image under α^* of some bundle $\widetilde{\zeta}'_r$. By Theorem 1.2,

$$\widetilde{A}_k \circ \rho_\omega^{S^1}(\alpha^\# \widetilde{\zeta}'_1, \ldots, \alpha^\# \widetilde{\zeta}'_{n-1}) = \alpha^* \widetilde{A}_k \circ \rho_\omega^{S^1}(\widetilde{\zeta}'_1, \ldots, \widetilde{\zeta}'_{n-1}),$$

and if $Q'_\omega(\eta)$ is a rational function whose expansion about 1 coincides with

$$\widetilde{A}_k \circ \rho_\omega^{S^1}(\widetilde{\zeta}'_1, \ldots, \widetilde{\zeta}'_{n-1}),$$

then $Q_\omega(\eta) = Q'_\omega(\eta^n)$. Since $Q'_\omega(\eta)$ does not have poles at 1, $Q_\omega(\eta)$ does not have poles at the nth roots of 1. Therefore the sum (2.4) is regular at $\exp(2\pi i p/n)$. If n is relatively prime with k, then from (2.2) it follows that at those points $\mathfrak{A}(\eta)$ is also regular. To prove the regularity of $\mathfrak{A}(\eta)$ at the remaining roots of 1 we shall use the following lemma.

LEMMA 2.3. *Assume that over a point in the fixed submanifold F_s the representation of S^1 in the fibers of the S^1-bundle ζ over X is equal to $\Sigma_{i=1}^n \eta^{j_{si}}$. If $c_1(\zeta)$ is divisible by k, then all the sums $\sigma_s = \Sigma_{i=1}^n j_{si}$ are congruent modulo k.*

PROOF. The action of S^1 on ζ naturally induces an action of S^1 on the determinant of ζ, a one-dimensional complex bundle ξ which is the lth exterior power of ζ, $l = \dim \zeta$. The representation of S^1 in the fibers of the S^1-bundle ξ over points of F_s is η^{σ_s}.

Now consider another action of S^1 on ξ. Since $c_1(\xi) = c_1(\zeta)$, we have $\xi = \kappa^k$, where the one-dimensional bundle κ is such that $c_1(\kappa) = (1/k)c_1(\xi)$. As was proved in [7], there exists a lifting of the action of S^1 on X to the total space of κ which turns it into an S^1-bundle. This action induces an action on the space of the representation ξ. If η^{δ_s} is the representation in the fibers of κ over points of F_s, then the representation in the fibers of ξ over them is $\eta^{k\delta_s}$.

If we compare the two actions of S^1 on ξ and remember that representations in the fibers of a one-dimensional bundle over fixed points are uniquely determined up to multiplication of all of them by η^N, we obtain

$$\sigma_s = k\delta_s + N.$$

This proves the lemma.

When the variable η is multiplied by $\theta = \exp(2\pi i/k)$, each term of (2.1) is multiplied by $\theta^{(\Sigma j_{ls} - \Sigma j_{ms})}$. Under the assumptions of the theorem, all the exponents $\Sigma_{j_{ls}} - \Sigma_{j_{ms}}$ are congruent modulo k, as follows from Lemma 2.3. Therefore there exists N such that $\mathfrak{A}(\theta\eta) = \theta^N \mathfrak{A}(\eta)$.

The rational functions $Q'_\omega(\eta)$ defined above have the same property. Thus, they are regular at all the points $\exp(2\pi iq/k)$, $0 \leqslant q \leqslant k-1$. The functions $Q_\omega(\eta)$ entering in sum (2.4) are regular at $\exp(2\pi iql/kn)$, $0 \leqslant l \leqslant n-1$. From (2.2) it finally follows that $\mathfrak{A}(\eta)$ is analytic at all the roots of 1, and this implies the assertion of the theorem.

THEOREM 2.2. *If the action of a compact connected Lie group G on a manifold X whose tangent bundle has first Chern class $c_1(X) \in H^2(X, Z)$ divisible by k is nontrivial, then*

$$A_k^G([X, G]) = 0;$$

in particular, $A_k([X]) = 0$.

PROOF. In the case when $G = S^1$, the theorem follows from the fact that, as it appears in the explicit formulas, each term of (2.1) tends to 0 when $\eta \longrightarrow 0$ and when $\eta \longrightarrow \infty$. So $\mathfrak{A}(0) = \mathfrak{A}(\infty) = 0$, and, by the preceding theorem, $\mathfrak{A}(\eta) \equiv 0$. For an arbitrary compact connected Lie group G we shall use Theorem 1.2 as follows. If $A_k^G([X, G]) \neq 0$, there is an embedding α: $S^1 \longrightarrow G$ such that $\alpha^* A_k^G([X, G]) \neq 0$; but this contradicts the equality $\alpha^* A_k^G([X, G])$ [illegible] ${}_k^{S^1}([X, S^1]) =$ [illegible].

2. *Orien*[illegible] *case.* As w[illegible]d in the Introduction, although for definiteness we only consider unita[illegible] structure, all the [illegible]esults can be obtained for the orientable case practically without changes. In particular if we note that the A_2-genus coincides with the classical A-genus and the hypothesis of Theorem 2.2 in the case $k = 2$ coincides with the assumption that X is a Spin-bundle, we obtain the following theorem.

THEOREM 2.2′. *If the action of a compact connected Lie group G on a Spin-bundle is nontrivial, then $A^G([X, G]) = 0$, and, in particular, its A-genus is equal to zero.*

This theorem was first proved in [8] by a fundamentally different method which utilized the Atiyah-Singer index theorem.

§3. Multiplicative genera of algebraic manifolds

The effectivity of any obstructions depends on the effectivity of the way they are computed. The language of formal group theory allows one to solve the problem of computing rational genera of algebraic manifolds and therefore also to find the values on them of the above-described system of obstructions to the existence of S^1-actions.

In [2] a formula for a generating series of bordism classes of hypersurfaces given on CP^n by an equation of mth degree was obtained (Theorem 4.11). The same method can be used to obtain formulas for bordism classes of an arbitrary algebraic manifold. However, in order to simultaneously obtain the results which will yield the proofs of the theorems in the following section, we shall use another way.

Assume that an algebraic manifold X_n is given by a system of homogeneous polynomials $P_{m_i}(x_0, \dots, x_n)$ of degree m_i, $1 \leq i \leq s$. Without losing generality we can assume that X_n is in general position with the submanifold CP^{s-1} given on CP^n by $x_0 = x_1 = \cdots = x_{n-s} = 0$, i.e. $X_n \cap CP^{s-1} = \varnothing$. Then the projection $p\colon CP^n \backslash CP^{s-1} - CP^{n-s}$, where CP^{n-s} is the submanifold having the last s homogeneous coordinates equal to zero, induces a map $\pi\colon X_n \longrightarrow CP^{n-s}$.

THEOREM 3.1. *The cobordism class* $\gamma(n; m_1, \dots, m_S) \in U^0(CP^{n-s})$ *dual to the bordism class* $[X_n, \pi] \in U_{2(n-s)}(CP^{n-s})$ *is equal to*

$$j^*_{n-s}\left(\prod_{i=1}^{s} \frac{[u]_{m_i}}{u}\right),$$

where $j_{n-s}\colon CP^{n-s} \longrightarrow CP^\infty$ *is the inclusion.*

PROOF. It follows immediately from the definitions that if j_{n_1, n_2} is the inclusion of CP^{n_1} into CP^{n_2}, $n_1 \leq n_2$, then

$$j^*_{n_1, n_2}\gamma\,(n_2 + l;\ m_1,\ \dots,\ m_s) = \gamma\,(n_1 + l;\ m_1,\ \dots,\ m_s).$$

Therefore there exists a bordism class $\gamma(m_1, \dots, m_s) \in U^0(CP^\infty)$ such that

$$j^*_{n-s}\gamma\,(m_1,\ \dots,\ m_s) = \gamma\,(n;\ m_1,\ \dots,\ m_s).$$

The space $CP^N \backslash CP^{s-1}$ is naturally identified with the total space $E(s\eta)$ of the sum of s copies of the canonical bundle over CP^{N-s}. Assume that $r\colon X_N \longrightarrow E(s\eta)$ is an embedding. Then $[X_N, \pi] = p_*([X_N, r])$, where the isomorphism $p_*\colon U_*(E(s\eta)) \longrightarrow U_*(CP^{N-s})$ is induced by p. The bordism class $[X_N, r]$ is equal to $p^*\gamma(N; m_1, \dots, m_s) \cap [CP^{N-s}, f_0]$, where $\cap$ is the Čech excision operator and f_0 is the zero cross-section. (This follows from the known formula $p_*(p^*(a) \cap b) = a \cap p_*(b)$.) Therefore the cobordism class dual to $[X_N, r]$ under the isomorphism $U_*(E(s\eta)) \longrightarrow \widetilde{U}^*(M(s\eta))$ is equal to $p^*\gamma(N; m_1, \dots, m_s) \cdot t(s\eta)$. As usual, $M(\ )$ and $t(\ )$ denote the Thom space and the Thom class of a bundle, respectively.

On the other hand, the class which is dual to the inclusion of X_N in CP^N is equal to the Euler class of the bundle $\Sigma_{i=1}^s \eta^{m_i}$. In fact, the total space $E\eta^m$ is identified with the $(N + 2)$-tuples of complex numbers $(x_0, \dots, x_N, y)$, $|\vec{x}|^2 = 1$, with the equivalence relation

$$(x_0,\ \dots,\ x_N, y) \sim (zx_0,\ \dots,\ zx_N, z^m y),\ z \in S^1.$$

The map $(x_0, \dots, x_N) \longrightarrow (x_0, \dots, x_N, \dots, P_{m_i}(x_0, \dots, x_N), \dots)$ correctly defines a cross-section of $\Sigma_{i=1}^s \eta^{m_i}$ which is transversal to the zero cross-section of CP^N along X_N. Therefore the image of the class dual to $[X_N, r]$ under the homomorphism $f_0^*\colon U^*(M(s\eta)) \longrightarrow U^*(CP^{N-s})$ is equal to $e(\Sigma_{i=1}^s \eta^{m_i})$.

Since $f_0^*(t(s\eta)) = e(s\eta)$, we arrive at the formula

$$\prod_{i=1}^{s} e\,(\eta^{m_i}) = \gamma\,(N, m_1,\ \dots,\ m_s) \cdot \prod_{i=1}^{s} e\,(\eta).$$

Finally,

$$\gamma(m_1, \ldots, m_s) = \prod_{i=1}^{s} \frac{[u]_{m_i}}{u},$$

and the theorem is proved.

For complex projective space CP^N, the bordism class dual to u^k is CP^{N-k}. Therefore if we denote by $CP(u)$ the series $\Sigma_{n=0}^{\infty} [CP^n] u^n$, the following corollary holds:

COROLLARY 1. *The bordism class of the manifold X_n given by a system of homogeneous polynomials $P_{m_i}(x_0, \ldots, x_n)$, $1 \leqslant i \leqslant s$, is equal to the coefficient of u^{n-s} in the series*

$$\left[\prod_{i=1}^{s} \frac{[u]_{m_i}}{u}\right] \cdot CP(u).$$

The series $CP(u)$ is equal to $dg(u)/du$. Therefore the value of a genus h on the bordism class of X_n is equal to

$$\frac{1}{2\pi i} \oint \frac{1}{u^{n-s+1}} \prod_{i=1}^{s} \frac{g_n^{-1}(m_i g_h(u))}{u} \cdot \frac{dg_h(u)}{du} du.$$

The integral is carried out on the circle $|u| = \epsilon$. By means of the usual substitution $u = g_h^{-1}(\ln t)$ we obtain

$$h([X_n]) = \frac{1}{2\pi i} \oint_{C'} [g_h^{-1}(\ln t)]^{-n-1} \cdot \prod_{i=1}^{s} g_h^{-1}(\ln t^{m_i})\, d\ln t. \tag{3.1}$$

The contour C' is the circle $|t - 1| = \epsilon$.

COROLLARY 2. *The value of A_k on $[X_n]$, where $n + 1 - \Sigma_{i=1}^{s} m_i = kd$, is equal to the coefficient of $(t-1)^n$ in the expansion in powers of $(t-1)$ of the function*

$$k^{n-s} t^{d-1} \prod_{i=1}^{s} (t^{m_i} - 1).$$

PROOF. Since $g_{A_k}^{-1}(\ln t) = (t^k - 1)/kt$, we have

$$A_k([X_n]) = \frac{1}{2\pi i} \oint_{C'} \left(\frac{kt}{t^k - 1}\right)^{n+1} \prod_{i=1}^{s} \frac{t^{km_i} - 1}{kt^{m_i}} d\ln t$$

$$= \frac{1}{2\pi i} \oint_{C'} \frac{k^{n+1-s} t^{kd}}{(t^k - 1)^{n+1}} \prod_{i=1}^{s} (t^{km_i} - 1)\, d\ln t$$

$$= \frac{k^{n-s}}{2\pi i} \oint_{C'} \frac{z^d}{(z-1)^{n+1}} \prod_{i=1}^{s} (z^{m_i} - 1)\, d\ln z,$$

where $z = t^k$.

If x is a generator of $H^2(CP^n, Z)$ and $j\colon X_n \longrightarrow CP^n$ is an embedding, then $c_1(X_n) = (n + 1 - \Sigma_{i=1}^{s} m_i) j^* x$. This corollary and Theorem 2.2 yield Theorem A, stated in the Introduction.

We shall state formulas for the $T_{x,y}$ genus of algebraic manifolds. The value of the $T_{x,y}$-genus on $[CP^n]$ is equal to $\Sigma_{i=0}^n x^i y^{n-i}$. If in (3.1) we set

$$g_{T_{x,y}}^{-1}(\ln t) = \frac{t^{x-y}-1}{xt^{x-y}-y}$$

and substitute z for t^{x-y}, we obtain

COROLLARY 3. *The value of the $T_{x,y}$ genus on the bordism class of an algebraic manifold X_n is the coefficient of $(z-1)^n$ in the expansion in powers of $z-1$ of the function*

$$\frac{1}{x-y}\cdot\frac{(xz-y)^{n+1}}{z}\prod_{i=1}^{s}\frac{z^{m_i}-1}{xz^{m_i}-y}.$$

REMARK. For $x = 1$ and $y = -1, 0$ we obtain the formulas for the signature and Todd genus of algebraic manifolds respectively.

§4. Bordism of ramified coverings

Assume that the finite cyclic group Z_n acts on a manifold Y. We shall assume that outside the set of fixed points, which is a submanifold (not necessarily connected) of real codimension 2, the action of Z_n is free. Then the quotient of Y by this action is a smooth manifold X, and the projection $p\colon Y \longrightarrow X$ is a ramified covering over X with ramification along the submanifold F of fixed points.

Our task is to compute both the bordism class of Y in terms of the bordism class of X and the invariants of the normal bundle ν over F, and the bordism class $[Y, p] \in U_*(X) \otimes Q$. (Here and in what follows F and $p(F)$ will be identified.) If $v \in U^2(X)$ is dual to the bordism class determined by the inclusion of F in X, we have

THEOREM 4.1. *The bordism class dual to $[Y, p]$ is equal to*

$$\frac{v}{g^{-1}(n^{-1}g(v))} \in U^0(X)\otimes Q.$$

PROOF. Let $p_1\colon Y_1 \longrightarrow X$ and $p_2\colon Y_2 \longrightarrow X$ be n-sheeted ramified coverings over X with ramification along F. Assume that the normal bundles of F in Y_1 and Y_2 are isomorphic; then the bordism class $[Y_1, p_1] - [Y_2, p_2]$ is equal to the class determined by the projection of the manifold $\widetilde{M}$ obtained by gluing together the complements N_1 and $-N_2$ of open tubular neighborhoods of F in Y_1 and Y_2 along an isomorphism of their boundaries.

The projections $p_1\colon N_1 \longrightarrow N$ and $p_2\colon -N_2 \longrightarrow N$, where N is the complement of a tubular neighborhood of F in X, induce a map $g\colon \widetilde{M} \longrightarrow M$ which is an n-sheeted covering. M is obtained by gluing N and $-N$ along the identity isomorphism of their boundaries.

LEMMA 4.1. *The bordism class $[\widetilde{M}, g] \in U_*(M) \otimes Q$ is equal to $n[M, \mathrm{id}]$, where* id *is the identity map of M into itself.*

PROOF. A bordism class is uniquely determined by the numbers $(c_\omega(\widetilde{M})g^*m, \langle\widetilde{M}\rangle)$, where $c_\omega(\widetilde{M}) = c_{i_1}(\widetilde{M})\cdots c_{i_r}(\widetilde{M})$, $\omega = (i_1, \ldots, i_r)$ and $c_i(\widetilde{M})$ are the Chern classes of the tangent bundle of $\widetilde{M}$; $m \in H^s(M)$ and $s = 2(i_1 + \cdots + i_r) = \dim M$. Here, as usual, $(a, \langle M\rangle)$ is the value of a cohomology class on the fundamental cycle.

Since g is a covering, we have the equalities

$$(c_\omega(\tilde{M})\cdot g^*m,\ \langle\tilde{M}\rangle)=(g^*(c_\omega(M)m),\ \langle\tilde{M}\rangle)=(c_\omega(M)m,\ g_*\langle\tilde{M}\rangle)$$
$$=n(c_\omega(M)m,\ \langle M\rangle),$$

whence the lemma follows.

Assume that $j\colon M\longrightarrow X$ is the natural map induced by the inclusion of N in X; then $[\tilde{M}, p]=j_*([\tilde{M}, g])=n([M, j])$. Since evidently $[M, j]$ is equal to zero, it follows that $[Y_1, p_1]=[Y_2, p_2]$.

To complete the proof of the theorem it is enough to do the following: for each bundle ξ over F such that $\xi^n=\nu$ we construct a ramified covering Y such that the normal bundle over F in Y is ξ, and we find the cobordism class dual to $[Y, p]$.

Assume that $f\colon X\longrightarrow CP^N$ is a map such that $f^*(\eta)=\xi$ (η is the canonical bundle over CP^N, where N is big enough). For an algebraic manofold X_N given by a homogeneous polynomial of degree n, which does not contain the point $(0, 0, \ldots, 0, 1)$, f^*X_N is well defined and is a ramified covering over X. In the notation of the preceding section, the class dual to $[f^*X_N, p]$ is equal to $f^*\gamma(N, n)$. The theorem follows from Theorem 3.1 and from the fact that $f^*(g^{-1}(ng(n)))=v$.

REMARK. Since ν is the nth tensor power of a bundle, the series with rational coefficients

$$\frac{1}{g^{-1}(n^{-1}(g(e(\nu))))}-\frac{n}{e(\nu)}=-\frac{CP^1}{2}(n-1)-\frac{CP^2}{3}\frac{n^2-1}{n}e(\nu)$$
$$-\frac{(CP^1)^2}{4}\frac{n^2-3n+3}{n^2}e(\nu)+\ldots$$

defines an element in the integral cobordism ring $U^*(F)$.

COROLLARY 1. *The cobordism class of a ramified covering Y is equal to*

$$n[X]+\varepsilon\left[\left(\frac{1}{g^{-1}(n^{-1}g(e(\nu)))}-\frac{n}{e(\nu)}\right)\cap F\right],$$

where $\epsilon\colon U_*(F)\longrightarrow U_*$ is the augmentation.

The formulas we have obtained allow us to find the values of rational Hirzebruch genera on bordism classes of ramified coverings. Thus, for example, we have

COROLLARY 2. *The value of $T_{x,y}$ on the bordism class of a ramified covering is*

$$nT_{x,y}([X])+\sum_{i=0}^{\infty}a_{i+1}T_{x,y}([e^i(\nu)\cap F]),$$

where a_i is the coefficient of t^i in the expansion in powers of t of the function

$$t\,\frac{x(1-ty)^{\frac{1}{n}}-y(1-tx)^{\frac{1}{n}}}{(1-ty)^{\frac{1}{n}}-(1-tx)^{\frac{1}{n}}}.$$

In several cases it is convenient to write the formulas for the bordism class of a ramified covering in terms of the invariants of the normal bundle over F in the ramified covering. Since $e(\nu)=g^{-1}(ng(e(\xi)))$, by Corollary 1 we have

COROLLARY 1$'$. *The bordism class of a ramified covering is equal to*

$$n([X])+\varepsilon\left[\left(\frac{1}{e(\xi)}-\frac{n}{g^{-1}(ng(e(\xi)))}\right)\cap F\right].$$

COROLLARY 2$'$ (HIRZEBRUCH) *If b_i is the coefficient of t^i in the expansion in power series of t of the function*

$$-t\,\frac{(1+t)^n+(1-t)^n}{(1+t)^n-(1-t)^n},$$

then

$$\operatorname{Sign}([Y])=n\operatorname{Sign}([X])+\sum_{i=0}^{\infty} b_{i+1}\operatorname{Sign}([e^i(\xi)\cap F]).$$

Received 26/MAR/75

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Translated by H. B. HERRERO