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On Generalized Bohr–Sommerfeld Quantization Rules for Operators with PT Symmetry

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Abstract—Bohr–Sommerfeld rules corresponding to quasi-eigenvalues in the spectrum for a one-dimensional h-pseudodifferential operator are given.

Key words: *pseudospectrum, Bohr–Sommerfeld rule, h-pseudodifferential operator.*

1. INTRODUCTION AND STATEMENT OF THE RESULT {s1}

Theorem 1. *Let P enjoy PT symmetry, and let p_0 be real.*

Proof. Using action-angle coordinates (s, τ), we obtain $p_0(s, \tau) = f_0(\tau)$. ... $\square$

.....

The letters and numerals in figures (EPS files) must be smaller by 1 point than in the main body of the paper.

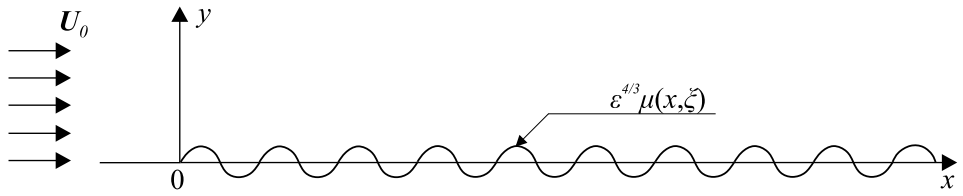

Fig. 1. The wave pattern Σ. {fig1}

Table 1 {tbl1}

Condition on k	$G(z)$	$H(z)$
$k \equiv 2 \,(\mathrm{mod}\, 12)$, $k \geq 14$	$E_4^2 E_6^{(k-14)/6}$	$E_4^{(k-2)/4}$
$k \equiv 4 \,(\mathrm{mod}\, 12)$, $k \geq 16$	$E_4 E_6^{(k-10)/6}$	$E_6 E_4^{(k-8)/4}$

Notes on formulas. Use \, to offset the differentials under the sign of the integral, e.g., $\int_A^B f(x,y)\,dx\,dy$, the commands \qquad to separate expressions or groups of formulas on a line, and \quad to separate formulas inside a group; also \; (or \ <space>) to separate subgroups inside a group. In a displayed formula, individual formulas must be ended by a comma or a period (full-stop), but not by a semicolon.

Do not use the "rough" commands "\left" and "\right" before any delimiters (parentheses, brackets). They often produce bad results. Before delimiters enclosing "big operators" (integrals, series, etc.) and fractions of standard form (\frac xy), use \biggl and \biggr. Fractions in lines of text and in exponents must only be used in *slashed form* (x/y). Also, for better "readability" of multiline displayed formulas, place the formatting commands (\label, \\, \nonumber, etc.) on separate lines, at their beginning (see the TEX file for this PDF).

Example. The Fourier series of a 2π-periodic function $f \in L_1(\mathbb{T})$, $\mathbb{T} = [-\pi, \pi]$, can be written as follows [1], [2]:

$$f \sim \sum_{k=-\infty}^{\infty} \widehat{f}_k e_k, \qquad e_k = e^{ikx}, \quad \widehat{f}_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-ikt}\, dt.$$

We also have

$$\begin{aligned} v(x,t) = -1 &- \int_0^{t-x} \int_0^{\tau/2} \biggl[r_0 v(\xi, \tau - \xi) - \int_0^{\tau - 2\xi} h(\alpha) v(\xi, \tau - \xi - \alpha)\, d\alpha \biggr] d\xi\, d\tau \\ &+ \int_{t-x}^{t} \int_{\tau - t + x}^{(2\tau - t + x)/2} \biggl[r_0 v(\xi, 2\tau - t - \xi) \sqrt{\frac{\tau - t}{2}}\, e^{-(\tau - t)/2} \\ &\qquad - \int_0^{2\tau - t + x - 2\xi} h(\alpha) v(\xi, 2\tau - t - \xi)\, d\alpha \biggr] d\xi\, d\tau \\ &< V(x,t) + A(x,t) + B(x,t) \qquad \text{for} \quad x > 1, \quad 0 < t < T, \ \ T > 1, \quad r_0 = 1, 2, \ldots . \end{aligned} \tag{1}$$

{eq1}

N.B. Mind the spacing in these formulas!!

For explicit rules and more examples (including the bibliography), see the files MN-RULES.pdf and MN-NEW_SAMPLE (TEX and PDF files).

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BIBLIOGRAPHY

1. M. Gusman, *Differentiation of Integrals in* $\mathbb{R}^n$ (Springer-Verlag, Heidelberg, 1975; Mir, Moscow, 1978).
2. A. S. Serdyuk and I. V. Sokolenko, "Asymptotic behavior of best approximations of classes of Poisson integrals of the functions of H_ω," J. Approx. Theory **163** (11), 1692–1706 (2011).
3. A. I. Shafarevich, "The behavior of the magnetic field in a conducting fluid with a rapidly varying velocity field," Dokl. Ross. Akad. Nauk **360**, 31–33 (1998) [Dokl. Math. **57** (3) 464–466 (1998)].
4. M. A. Krasnosel'skii and Ya. B. Rutitskii, *Convex Functions and Orlicz Spaces*, in *Contemporary Problems of Mathematics* (Fizmatgiz, Moscow, 1958), Vol. 1 [in Russian].

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